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A Hamiltonian Framework for Kinodynamic Planning and Control

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Abstract

Control based on artificial potentials results in bounded-torque controllers. We extend this notion by devising energy-reference controllers that use harmonic potentials to produce collision-free kinodynamic behavior. In addition, the phase space structure of a manipulator controlled by an artificial potential "copies" the critical point structure of that potential. This result is used in conjunction with harmonic potentials to generate and control convergent and repetitive motions for a manipulator.

1 Introduction

Koditschek [1] introduced the formal notion of an admissible potential function which is suitable for robot path planning. A series of papers [2, 3, 4] established a framework for using artificial potentials to plan and control the trajectories of a mechanical system (e.g. a robot manipulator). Using a total energy formulation, bounded-torque "safe" controllers can be derived as long as the artificial potential satisfies certain constraints. In [2] it is shown that the manipulator copies the critical point structure of the artificial potential.

In this paper, we assume a harmonic potential, which generally satisfies the constraints established by Koditschek, and has independently been proposed as a robot path planning mechanism [5]. Harmonic potentials offer two chief advantages:

1. They obey the min-max property, and thus exhibit no local extrema away from goal and obstacles,
2. They can be computed using resistive networks [6, 7].

Using the latter approach, these potentials can be computed over arbitrary environments, if they are discretized. Computation is extremely fast in this case. Even on a digital computer, relaxation times for computing potentials in the cases shown here were in the millisecond range. Detailed discussion of this method can be found in [5].

Ideally, the approach presented here generates constant energy *orbits* when released from an initial state with non-zero potential. In practice, dissipative forces cause such orbits to erode over time, and harmonic functions are not generally convergent at goal states. The regulation of such orbits can be achieved by using an energy-referenced feedback compensator based on

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an artificial Hamiltonian function for the system. A reference energy of 0 causes the system to converge to one of the minimum points of the potential and positive energy references induce orbits around stable critical points of the potential.

The use of a constant-energy constraint for control has been explored for hopping and juggling robots [8, 9, 10]. The "energy reference" concept described here is likewise an attempt to control the system such that the system's total energy is conserved. In the case of hopping or juggling robots, all of the terms involved in computing total energy arise from natural, possibly external constraints (e.g., spring energy, gravity, contact damping). In contrast, the treatment here relies on an artificial potential (in the same sense as in [3]).

In addition, a recent neurophysiological model [11, 12] postulates a role for the basal ganglia as a potential-based motor planning mechanism. The work described here offers a plausible extension of this biological model into the realm of continuous and repetitive motion.

2 Energy-Reference Control

We will consider the class of controllers consisting of a model-reference controller whose command accelerations were obtained from the gradient of an artificial potential. We will assume that the unit mass system has the following artificial Hamiltonian function:

$$H(q,p) = \frac{1}{2}||p||^2 + \phi(q) \tag{1}$$

where ϕ is an artificial potential, and q and p are configuration and momentum, respectively. This results in the following phase space trajectory constraints:

$$\begin{aligned} \dot{q} = & \quad \frac{\partial H}{\partial p} & = p \\ \dot{p} = & \quad -\frac{\partial H}{\partial q} & = -\nabla\phi \end{aligned} \tag{2}$$

Note that Equation 1 is essentially the total energy function for the system. Thus, the reference acceleration is proportional to the negative gradient of the potential ϕ. In this paper, we assume ϕ is harmonic (see [5]).

In practice, an energy-referenced damper is used to regulate system energy. This can be used to overcome dissipative forces during orbital motion ($H_{ref} = E_0$), or can be used to dissipate energy during convergent motion ($H_{ref} = 0$). Figure 1 shows the proposed system. Ideally, the total energy of the system should remain at a reference energy:

$$H_{ref} = \frac{1}{2}(p + \Delta p)^2 + \phi(q) \tag{3}$$

where q and p are the configuration and momentum, respectively, and Δp is the momentum change required to satisfy the energy economy.

$$\Delta p = [2(H_{ref} - \phi(q))]^{1/2} - p. \tag{4}$$

For a given reference energy, H_{ref} in Figure 1, this compensator is $\psi(\phi(q), p)$ defined as follows:

$$\begin{aligned} \psi(\phi(q),p) &= [2(H_{ref} - \phi(q))]^{1/2} - p \\ & \qquad when \ \ ((H_{ref} - \phi(q)) > 0) \\ &= -p \qquad otherwise \end{aligned} \tag{5}$$

In the first case, $H_{ref} > \phi(q)$, and enough energy is introduced in the form of momentum to make up for the system energy deficit. In the second case, the system potential, $\phi(q)$, is already too high with respect to the reference and the kinetic energy compensation behaves as a damper to dissipate momentum. In either case, kinetic energy is added or removed only along the current trajectory. A variety of other possibilities for redistributing kinetic energy are discussed in Section 7.

Therefore, the reference acceleration of the system is:

$$\ddot{q}_{ref} = -\nabla\phi \ + \ G_{KE} \ \psi(\phi(q), p) \tag{6}$$

where G_{KE} is the derivative gain.

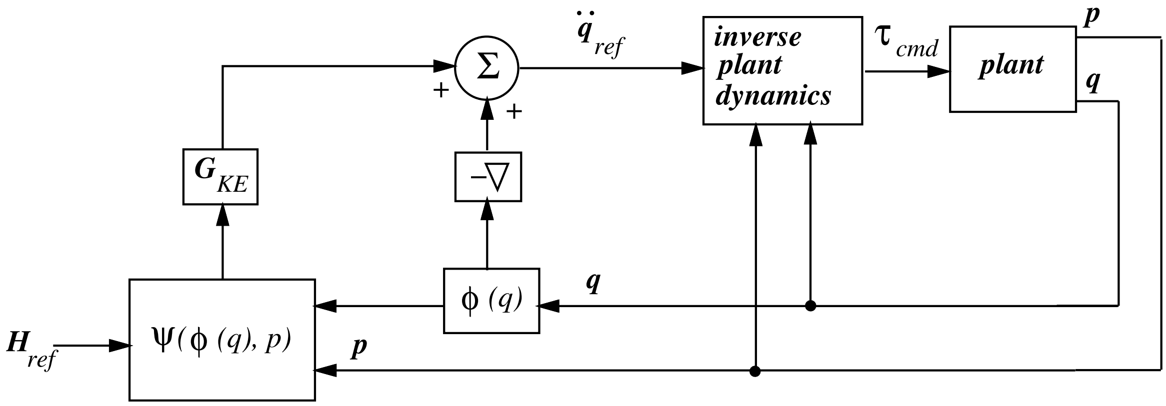

Figure 1: Energy-reference controller.

3 Critical Points

An artificial potential ϕ is used to plan the motion of a manipulator, and command torques are generated by using the gradient $\nabla\phi$ as an acceleration command for the servo loop. For a model-reference controller, the following equations of motion apply (assuming no external forces):

$$\tau = M\ddot{q} + V(q, \dot{q}) + G \tag{7}$$

Rewriting in terms of $\ddot{q}$:

$$\ddot{q} = M^{-1}(-V(q, \dot{q}) - G + \tau) \tag{8}$$

Command torques τ_{cmd} can be computed by applying Equation 7 to the command acceleration $-\nabla\phi$. The resulting equations of motion are:

$$\ddot{q} = -\nabla\phi \tag{9}$$

The fixed points of Equation 3 are those points where the velocity and potential gradient are simultaneously zero. Thus, the controller "copies" the critical points of ϕ into the phase space for the system. If ϕ is harmonic, it satisfies the min-max principle: In any neighborhood of a point, the minimum and maximum values of ϕ are found on the boundary of that neighborhood [13]. Hence, harmonic functions satisfy the conditions of Proposition 3.2 in [2], i.e. their level sets define sets of bounded total energy for the controlled mechanical system.

4 Kinodynamic Planning

Obstacle potentials can be fixed at a uniform value, ϕ_{obs}, which is the maximum potential energy in the system. Under an energy-referenced control scheme, the system energy will never exceed the obstacle potential. If errors in the energy-reference controller are neglected, then as long as $H_{ref} < \phi_{obs}$, the system can not encounter a configuration space obstacle with non-zero velocity. In general, equipotential sets of ϕ at $\phi = H_{ref}$ will bound the motion of the system.

In this paper, kinetic and potential energy are traded off along the current trajectory. Other constraints on momentum are possible, however. In this sense, the scheme described here is a form of kinodynamic planning, where the physical behavior of the manipulator can be modified by changing the momentum vector, and not simply its magnitude.

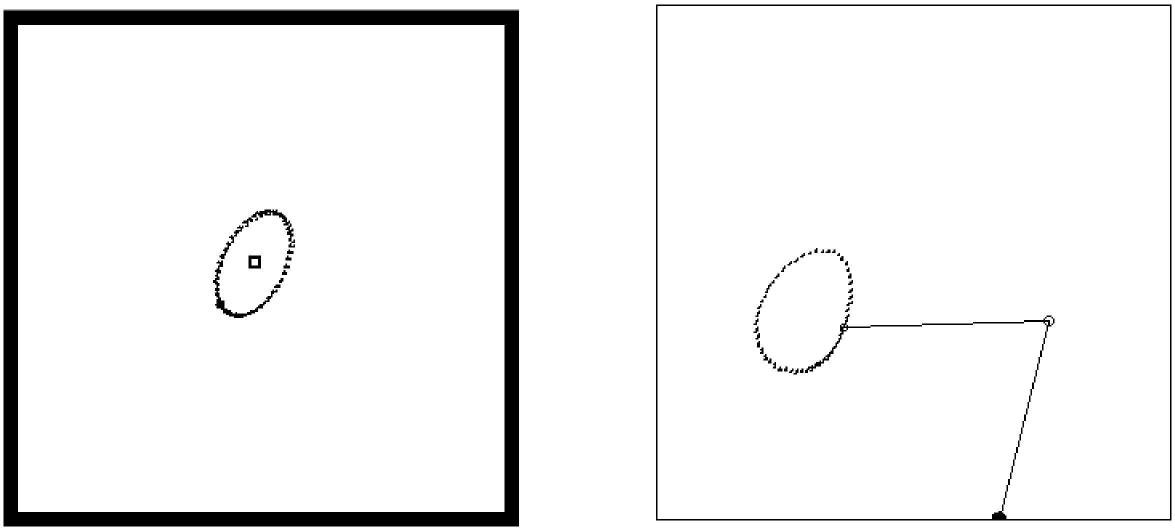

Figure 2: Constant-energy orbit. Left: configuration space; Right: cartesian space.

5 Resonance

The system can be driven to an oscillating or resonating state by setting H_{ref} to some value between 0 and ϕ_{obs}. This results in a constant energy trajectory that is bounded by the corresponding equipotential set of ϕ. Figure 2 shows an example of such an orbit, using a dynamic simulation of a 2-link revolute arm. The potential minimum is seen as an open square in configuration space (leftmost pane), while the filled squares on the boundary of the left pane are obstacles at the maximum potential.

The balance of kinetic and potential energy maintains the elliptical motion shown in Figure 2. Since the structure of the phase space is determined entirely by the critical point structure of the potential, it is possible that this representation can be used to "plan" simple repetitive motions. When H_{ref} is small enough, the system is trapped within the basin of attraction of a goal (low-potential) point, and will orbit that point indefinitely.

6 Implementation

A VME-based open-torque controller was used to implement this control scheme for a GE-P50 robotic arm. For these experiments, the shoulder and the elbow were controlled forming a 2-dof planar arm. Figure 3 shows an example of an orbit on the GE-P50 arm. The left pane depicts the configuration space of the robot where the cross represents the minimum potential value of 0 and the squares represent the maximum potential value of 1. The right pane shows the different energy levels measured during the orbit.

7 Other Issues

Several other issues arise from this approach: The constant energy manifold in a 4-dimensional phase space is 3 dimensional, and is underconstrained. By distributing the feedback energy appropriately, various constraints can be expressed in phase space, e.g., maintenance of a particular phase relationship between joint velocities. This appears to be one way of enforcing the eccentricity of the orbit. It is also possible in many cases to enforce certain minimum

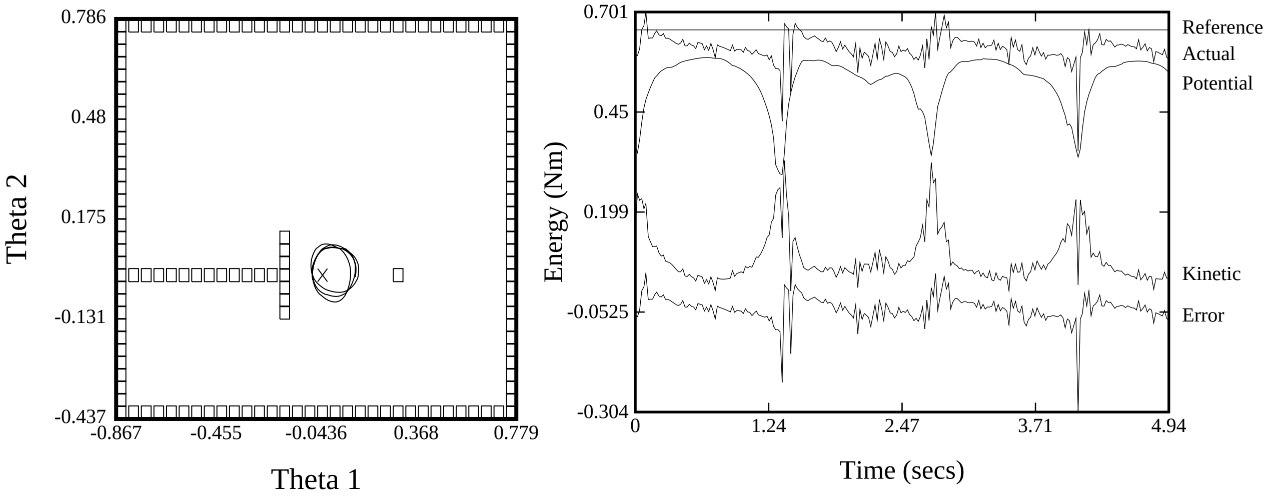

Figure 3: GE-P50 constant-energy orbit. Left: configuration space; right: energy levels.

and maximum bounds on a *subset* of phase variables in certain regions of configuration space.

The system as described is a perturbed Hamiltonian system, where the feedback law provides perturbations to $\dot{p}$ at servo rate. If there is a single minimum within one simply connected component of the configuration space, then we expect the trajectory to be fairly stable (although underconstrained). When there is more than one minimum, however, it might be expected that stochastic regions would develop near the hyperbolic fixed points in phase space, arising from saddles in ϕ. A more detailed analysis of the perturbation could yield energy or action-variable limits that would guarantee stability.

Finally, harmonic functions can be deformed in non-homogeneous resistive grids or by asserting new boundary constraints. These mechanisms are being evaluated as a means of programming the shape of the level sets of the energy function *without* introducing new critical points.

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