Optimal Multifingered Grasp Synthesis *

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Abstract

This paper discusses how grasp synthesis can be cast as a control composition problem. Two elemental grasp controllers are presented and composed into a general purpose grasp controller through the use of standard optimal control techniques. The resulting grasp synthesis procedure is scalable on the number of contacts, is not tied to any specific object representation scheme, and can incorporate kinematic constraints into its resolution.

1 Introduction

Grasp synthesis, as idealized by most researchers, consists in the off-line computation of a grasp configuration that maximizes a certain grasp metric the author is interested in [2, 6, 7, 9, 10, 11]. Typically, a complete model [6, 7, 11] or a smooth approximation [2, 9] of the object geometry is assumed. However, the synthesis procedure is computationally expensive [6] or it may involve exhaustive search over the object geometry [7, 11]. Some approaches restrict the number of contacts according to the object dimensionality (2 contacts in 2D and 3D [2], 2 contacts in 2D [6], at least 3 contacts in 3D and 2 contacts in 2D [10]), others constrain the object representation (polygons or polyhedra are assumed in [7, 11], and smooth/parametric models are required in [2, 6, 9]).

Alternatively, grasp synthesis can be cast as a control composition problem. The underlying assumption is that a controller for executing wrench closure on general convex geometries can be constructed by composing control modules drawn from a set of basis controllers. This paper describes how optimal control techniques can be used to solve the control composition problem. The resulting grasp synthesis procedure is not tied to any specific object representation scheme, is applicable to both 2- and 3-dimensional grasp synthesis, and is computationally efficient with respect to the number of contacts. It also can be used to incorporate constraints related to grasp execution (e.g., manipulator kinematics) into the synthesis process.

The next section presents the elemental grasp controllers that constitute the basis for the grasp synthesis procedure, followed by the formulation of control composition as an optimal control problem. A simple grasp synthesis problem is then used to illustrate our approach.

2 Grasp controllers

Grasp synthesis procedures are designed around a grasp metric which is maximized during the synthesis process. Grasp metrics usually emphasize one or more aspects of the resulting grasp configuration: capacity to resisting external disturbances [9], distance between contacts [2], ratio between resulting wrenches and applied forces, or minimization of total forces applied to the object [7], grasp stability [10], and others.

A grasp configuration is stable if it can resist arbitrary perturbation wrenches. The approach taken observes that homogeneous grasp solutions based on frictionless point contacts models can exploit whatever real contact friction exists to dilate the contact wrench envelope and therefore improve grasp robustness. Moreover, such grasp geometries can be used to squeeze the object within the "frictionless null space" to scale the size of the friction dilated wrench envelope.

This section describes two controllers, respectively the force closure (FC) controller and the moment closure (MC) controller, designed to maximize grasp stability. Both controllers are based on estimates of the local wrench closure error surface. The FC controller derives its estimate on the basis of relative contact position alone, while the MC controller employs positions and normals to the contact surfaces. Contacts are assumed to be frictionless point contacts and map to forces and moments $\in [0,1]$ with respect to the object frame.

2.1 Stable grasp sufficiency metric

In scoring a grasp configuration, it is useful to analyze the associated grasp matrix (also known as the grasp Jacobian). This matrix describes the transformation from a set of forces applied by contacts on the object surface to a set of object frame wrenches. The grasp matrix W is defined as $W = [W_1\, W_2\, \ldots W_n]$, where W_i is a six dimensional vector of wrenches applied at the i^{th} contact position.

The null space of the frictionless grasp matrix defines a basis for a set of equivalent grasp geometries. However, the procedure for computing the null space of a matrix does not allow for differentiability and therefore cannot be used as a control surface. The same shortcomings plague other grasp metrics: they are computationally expensive and frequently involve optimization procedures or matrix analysis techniques that do not offer directional information on which a *grasp controller* can be based.

To overcome those shortcomings, a sufficiency metric was devised [8], based on the residual wrench vector $\vec{\rho}$

$$\begin{aligned}\epsilon &= \vec{\rho}^T \vec{\rho} \\ &= \left(\vec{t} - \frac{1}{n}\sum_{i=1}^{n}\hat{\omega}_i\right)^T \left(\vec{t} - \frac{1}{n}\sum_{i=1}^{n}\hat{\omega}_i\right), \quad (1)\end{aligned}$$

where $\vec{\rho}$ expresses the net wrench over $1 \leq j \leq n$ contacts, $\vec{t}$ is a user-specified wrench closure *bias*[1], and $\vec{\omega}_i$ is the wrench vector resulting from the i^{th} interaction force. The elements $t_j \in [-1,1]$ of $\vec{t}$ and the elements $w_{ij} \in [-1,1]$ of $\vec{w}_i$ are qualitative in the sense that they do not reflect engineering units of force and torque, but express the relative ability of a contact configuration to transmit forces and torques through the object's surface. Minima in ϵ correspond to linear combinations of these qualitative wrenches which map into the grasp matrix's null space.

By moving the contacts in directions that minimize ϵ, the grasp controller approaches the desired null grasp configuration. One plausible approach is to compute the gradient of ϵ with respect to the contact coordinates θ_i and move the contacts accordingly:

$$\begin{aligned}\frac{\partial \epsilon}{\partial \theta_i} &= 2\left(\vec{t} - \frac{1}{n}\sum_i \hat{\omega}_i\right)^T \frac{\partial\left(\vec{t} - \frac{1}{n}\sum_i \hat{\omega}_i\right)}{\partial \theta_i} \\ &= -2\left(\vec{t} - \frac{1}{n}\sum_i \hat{\omega}_i\right)^T \frac{\partial \omega_i}{\partial \theta_i}\frac{1}{n} \\ &= -\frac{2}{n}\vec{\rho}^T G_i \\ &= -\frac{2}{n} G_i^T \vec{\rho}, \quad (2)\end{aligned}$$

[1] For the null grasp task, $\vec{t} = \vec{0}$.

where G_i is the wrench gradient with respect to the contact coordinate θ_i. In vectorial notation

$$\begin{aligned}\frac{\partial \epsilon}{\partial \vec{\theta}} &= -\frac{2}{n}\begin{bmatrix} G_1^T \\ G_2^T \\ \vdots \\ G_n^T \end{bmatrix} \vec{\rho} \\ &= -\frac{2}{n} G^T \vec{\rho}. \quad (3)\end{aligned}$$

Equation 3 evaluates to $\vec{0}$ in one of two conditions: (a) $\vec{\rho} = \vec{0}$ and (b) when $G^T\vec{\rho} = \vec{0}$. The first condition is the desired convergence criterion and can be used to suppress external perturbations by scaling grasp wrenches to span the task. The second condition is equivalent to a local minimum of ϵ. This class of minima results when a local tangent to the wrench surface, G, is orthogonal to the residual vector, $\vec{\rho}$. Both grasp controllers are based on Equation 3: what distinguishes them is exactly how $\hat{\omega}_i$ varies with the contact coordinate, θ_i.

2.2 Force closure controller

The force closure (FC) controller is used to navigate frictionless point contacts on the continuous Gaussian sphere. This simplified system uses only relative contact position to eliminate the force closure component of the residual (Equation 1). If θ and ϕ are the angular coordinates on the Gaussian sphere, then the corresponding wrench domain model of the object $\vec{W}(\theta,\phi) = [F_x\ F_y\ F_z\ M_x\ M_y\ M_z]$ becomes

$$\begin{aligned} w_1 &= f_x = -cos(\theta)cos(\phi) & w_4 &= m_x = 0 \\ w_2 &= f_y = -sin(\theta)cos(\phi) & w_5 &= m_y = 0 \\ w_3 &= f_z = -sin(\phi) & w_6 &= m_z = 0 \end{aligned}$$

The sufficiency metric for the FC controller yields:

$$\begin{aligned}\epsilon_{FC} = &\left[t_{fx} + \frac{1}{n}\sum_{i=1}^{n} cos(\theta_i)cos(\phi_i)\right]^2 + \\ &\left[t_{fy} + \frac{1}{n}\sum_{i=1}^{n} sin(\theta_i)cos(\phi_i)\right]^2 + \\ &\left[t_{fz} + \frac{1}{n}\sum_{i=1}^{n} sin(\phi_i)\right]^2 \quad (4)\end{aligned}$$

The gradient of Equation 4 with respect to θ_k yields:

$$\frac{\partial \epsilon_{FC}}{\partial \theta_k} = (A_{\theta k}^2 + B_{\theta k}^2)^{1/2} sin(\theta_k + \psi_k) \quad (5)$$

where $A_{\theta k}$, $B_{\theta k}$, and ψ_k are constants independent of θ_k:

$$\begin{aligned} A_{\theta k} &= \frac{2}{n}\left(t_{fy} + \frac{1}{n}\sum_{i \neq k}(sin(\theta_i)cos(\phi_i))\right) cos(\phi_k) \\ B_{\theta k} &= -\frac{2}{n}\left(t_{fx} + \frac{1}{n}\sum_{i \neq k}(cos(\theta_i)cos(\phi_i))\right) cos(\phi_k) \\ \psi_k &= tan^{-1}(A_{\theta k}/B_{\theta k}) \end{aligned}$$

Equation 5 demonstrates that the sufficiency metric employing the force closure model generates a unimodal error function.

2.3 Moment closure controller

Reasoning in terms of surface curvature, a scaled version of the FC controller can capture the local shape of the sufficiency metric for all positive and negative curvatures: the scaling constant is proportional to $\frac{1}{r}$, where r is the radius of curvature. However, as $r \to \infty$ the FC gradient vanishes, since forces remain constant on planar surface facets. The moment closure (MC) controller minimizes the moment residual by adjusting each contact position on the local surface facet. We use the perpendicular of a plane, (r_O, θ_O, ϕ_O), to parameterize the contact plane. Figure 1 illustrates the parameterization and the geometry from which the wrench gradient is derived.

The forces transmitted through any planar face are constant at all contact positions on that face. The moments applied to the object, $\vec{m} = \vec{r} \times \vec{f}$, vary linearly with surface coordinate and pass through zero where the perpendicular passes through the plane. The resulting wrench model is:

$$\begin{aligned} w_1 &= f_x = -cos(\theta_O)cos(\phi_O) \\ w_2 &= f_y = -sin(\theta_O)cos(\phi_O) \\ w_3 &= f_z = -sin(\phi_O) \\ w_4 &= m_x = -r_\theta sin(\phi_O)cos(\theta_O) + r_\phi sin(\theta_O) \\ w_5 &= m_y = -r_\theta sin(\phi_O)sin(\theta_O) - r_\phi cos(\theta_O) \\ w_6 &= m_z = r_\theta cos(\phi_O) \end{aligned} \quad (6)$$

where $(\hat{r_\theta}, \hat{r_\phi})$ are the surface coordinates of the contact in the plane. Note that this approximation yields constant contact forces which implies that control is derived from the moment diagrams exclusively; thus the designation "moment closure". The moment closure sufficiency metric can be written as follows:

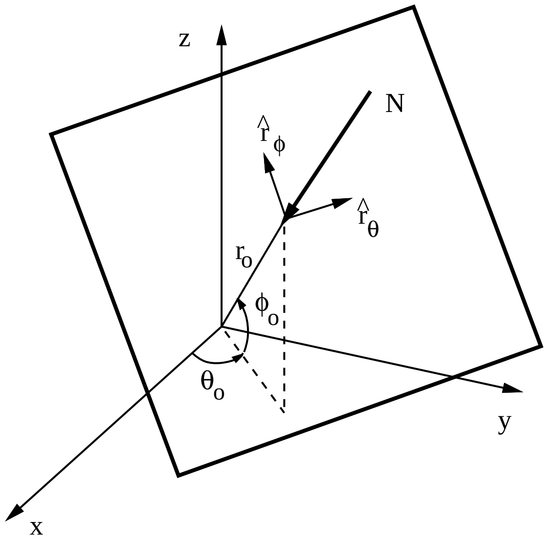

Figure 1: The Moment Closure Model derived from a Parametric Object Plane

$$\begin{aligned} \epsilon_{MC} &= \sum_i \epsilon_{MC}^{w_i}, \quad (7) \\ \epsilon_{MC}^{w_i} &= \left[t_{w_i} - \frac{1}{n}\sum_i w_i\right]^2 \end{aligned}$$

Figure 2(a) shows the plot of both the force closure metric (Equation 4, dashed curve) and the plot for the sufficiency metric (Equation 1, solid curve), for the square shape, 2 contacts. Those plots are obtained by fixing the black contact at the origin $\theta = 0$, and moving the white contact counterclockwise over the object surface. Figure 2(b) shows the complete object model; in the same figure, the sufficiency metric's local minima are marked over the object surface with triangles, and the white contact is positioned over the metric's global minimum. Figure 2(c) shows the corresponding constant curvature unit Gaussian sphere, and its global (and only) minimum.

Figure 2 illustrates how the FC controller may contribute to grasp synthesis: it is a global approximation of every manifold geometry with no local minima. This heuristic control surface selects combinations of faces that minimize the force closure residual.

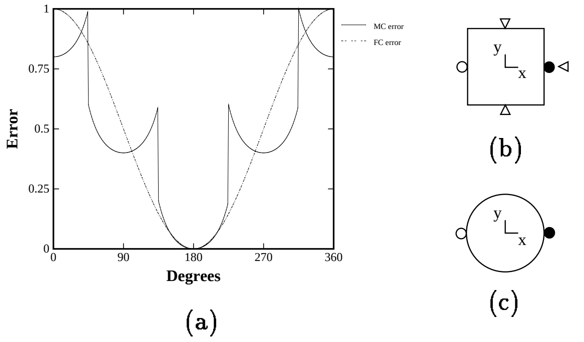

Figure 2: (a) Sufficiency metric error plot, for the complete object model (solid curve) and for the smoothed object model (dashed curve). (b) The triangles mark the position of the sufficiency metric local minima on the object surface. (c) FC object model. In this case, the global minima for both metrics coincided on $\theta = 180^o$.

2.4 Controller properties

The FC controller is based on contact positions exclusively; the contacts navigate the surface of the continuous Gaussian sphere, and the resulting controller can be made globally convex [4]. The MC controller considers both contact position and normal; the resulting control surface is piecewise convex. Each controller encodes a model of how local contact geometry transforms contact forces into contact wrenches, but no explicit model of the object itself is ever required – only local sensory information. Moreover, it is possible to assign an independent asynchronous controller to each contact. As a consequence, the complexity of the composite controller grows linearly with the number of contacts.

In [4, 5], knowledge-based heuristic rules, learning techniques, and pre-imaging control are used in the composition of the FC and MC controllers. These techniques do not require a complete model of the object; control actions can be computed directly from sensor readings of contact positions and normals. Although this is probably the most adequate paradigm for the control of real-world complex systems, one can do better if complete or very good models about the problem is available, using optimal control techniques. Portraying the composition of controllers as an optimal control problem illustrates a general solution strategy that can be used in other domains to solve complex control problems.

3 Control composition as an optimal control problem

A small and somewhat constrained version of the grasp synthesis problem was chosen to illustrate our approach, namely the null grasp task of planar, convex objects with 2 contacts. There is no theoretical impediment in generalizing this technique to multi-contact grasp of 3D objects. For the general control composition problem, the only requirements are continuity of the metric being optimized, and differentiability of the constituent controllers. The problem

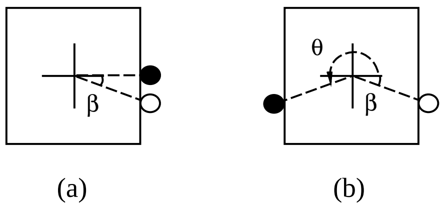

Figure 3: (a) Initial configuration (b) Final configuration.

addressed in this paper consists of activating the FC and MC controllers, so as to minimize the number of control steps required to take the system from a initial to a final (unknown) configuration (see Figure 3). For simplicity, we will consider that just one contact moves over the object surface, while the other stays fixed at its initial position. Also, we will assume that the activation for the MC controller is constrained to be $(1-\alpha)$, where $\alpha \in [0,1]$ is the activation for the FC controller.

3.1 Problem formulation

Maximize

$$V = \int_0^T -\epsilon_{MC}(\theta)\, dt$$

Subject to

$$\begin{aligned} \dot{\theta}(t) &= -\left(\alpha \frac{d\,\epsilon_{FC}(\theta)}{d\,\theta} + (1-\alpha)\frac{d\,\epsilon_{MC}(\theta)}{d\,\theta}\right) \\ \theta(0) &= 0.0 \\ \alpha &\in [0,1] \end{aligned}$$

In the formulation above, ϵ_{MC} is the wrench closure error (Equation 7). For two contact grasps of square geometries, $\epsilon_{MC}(\theta)$ can be written as

$$\epsilon_{MC}(\theta) = E + \frac{(2\tan\theta - 1)^2}{16},$$

where E is a constant corresponding to the force error incurred in positioning the moving contact in the various faces of the square. For the face corresponding to $\theta = 0$, $E = 1$; for faces corresponding to $\theta = \pi/2$ and $\theta = 3\pi/2$, $E = 0.5$, and for the face corresponding to $\theta = \pi$, $E = 0$.

Similarly, $\epsilon_{FC}(\theta)$ is the force closure error, expressed as

$$\epsilon_{FC}(\theta) = 0.5 * (1 + \cos(\theta - \beta)),$$

being β the angle for the fixed contact. In the development that follows, the first and second derivatives of $\epsilon_{MC}(\theta)$ and $\epsilon_{FC}(\theta)$ with respect to θ will be required:

$$\begin{aligned} \frac{d\,\epsilon_{MC}}{d\,\theta} &= \frac{2\tan\theta - 1}{4\cos^2\theta} \\ \frac{d^2\epsilon_{MC}}{d\,\theta^2} &= \frac{8 + 2(\tan\theta - 1)\sin 2\theta}{16\cos^4\theta} \\ \frac{d\,\epsilon_{FC}}{d\,\theta} &= -0.5\sin(\theta - \beta) \\ \frac{d^2\epsilon_{FC}}{d\,\theta^2} &= -0.5\cos(\theta - \beta) \end{aligned}$$

The Hamiltonian function for the problem can now be expressed as

$$H(t, \theta, \alpha, \lambda) = -\epsilon_{MC}(\theta) + \lambda(t)\frac{d\,\theta}{d\,t}.$$

The costate variable λ is the Lagrangian multiplier required by the method [1, 3]. The Maximum Principle conditions are

$$H(t, \theta, \alpha^*, \lambda) \;\geq\; H(t, \theta, \alpha, \lambda) \;\; \forall t \in [0, T] \qquad (8)$$

$$\begin{aligned} \dot\theta(t) &= \frac{\partial H}{\partial \lambda} \\ &= -\left(\alpha\frac{d\,\epsilon_{FC}(\theta)}{d\,\theta} + (1-\alpha)\frac{d\,\epsilon_{WC}(\theta)}{d\,\theta}\right) \quad (9) \\ \dot\lambda(t) &= -\frac{\partial H}{\partial \theta} = \frac{d\,\epsilon_{MC}}{d\,\theta} \\ &+ \lambda(t)(\alpha\frac{d^2\epsilon_{FC}}{d\,\theta^2} + (1-\alpha)\frac{d^2\epsilon_{MC}}{d\,\theta^2}) \qquad (10) \\ \lambda(T) &= 0 \end{aligned}$$

Equation 8 means that the resulting Hamiltonian H is to be maximized over time, and $\alpha^*(t)$ is the activation policy that maximizes it. Notice that selecting α such that $\frac{\partial H}{\partial \alpha} = 0$ does not cover all cases, because of the constrained range of values α can assume — it might be the case that α^* corresponds to one value in the limits of the closed interval $[0, 1]$.

This is the case for the problem we are interested in; for this example, $\frac{\partial H}{\partial \alpha}$ does not depend on α:

$$\frac{\partial H}{\partial \alpha} = -\lambda(\frac{d\,\epsilon_{FC}}{d\,\theta} - \frac{d\,\epsilon_{MC}}{d\,\theta}).$$

Therefore, the time optimal policy for this particular problem implies switching α from 0 to 1 and back. This optimal pattern can be easily computed by integrating Equations 9 and 10 above over time:

1. Initialize λ;
2. Compute $\frac{\partial H}{\partial \alpha}$; if $\frac{\partial H}{\partial \alpha} > 0$, select $\alpha = 1$; otherwise, make $\alpha = 0$;
3. Compute $\theta_{t+1} = \dot\theta_t \Delta t$;
4. Compute $\lambda_{t+1} = \dot\lambda_t \Delta t$;
5. Check for convergence on θ; return to (2) above if not converged.

4 Results

In all results, we used $\lambda(0) = 5$, and $\Delta t = 0.01$. The method is not sensitive to the values chosen for these parameters; an ample range of values can be used. However, selecting a low initial value for λ may result in solutions corresponding to local minima. The fixed contact is stationed at $\beta = -0.46365$ radians.

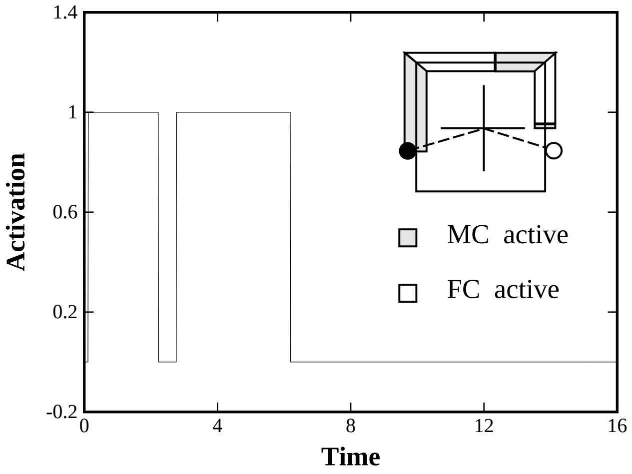

Figure 4: Time optimal activation pattern during the grasp. $\alpha = 1$ corresponds to full activation of the FC controller, while $\alpha = 0$ corresponds to full activation of the MC controller.

Figure 4 shows the optimal activation pattern for α as a function of time. Figure 4 also displays the corresponding areas on the object surface where the controllers were selected: gray areas correspond to where in the object the MC controller was selected ($\alpha = 0$), and white areas label the those areas in which the FC controller received full activation ($\alpha = 1$).

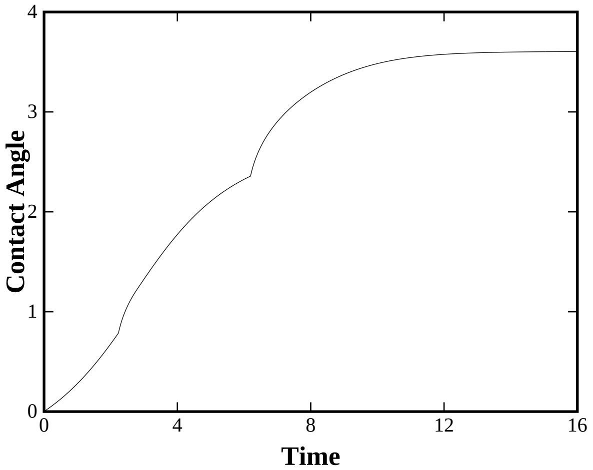

Figure 5: Evolution of θ with time.

Figure 5 shows how θ varies with time. Starting from $\theta(0) = 0.0$, the contact progresses steadily to the optimal solution $\theta(T) = 3.6052$ radians. First-order discontinuities are present whenever the moving contact changes faces on the square.

5 Conclusion

As long as the metric chosen to be optimized doesn't depend on the intermediate activations $\alpha(t)$, the time optimal activation policy will always be a *bang-bang* type solution, no matter the object being grasped.

Another interesting aspect is the fact that this approach also yields an effective grasp controller: just by evaluating $\frac{\partial H}{\partial \alpha}$ one can decide on the next activation α. What remains to be determined is how robust this controller is with respect to object geometries, and uncertain actuation and sensory information. Because this approach essentially involves integration over time, it is not clear how to revise the control history incrementally: presumably, a object model is being continuously or periodically updated, and sensory data may yield discontinuities in the state information. How to accommodate such changes in the control scheme is yet to be determined.

Acknowledgements

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