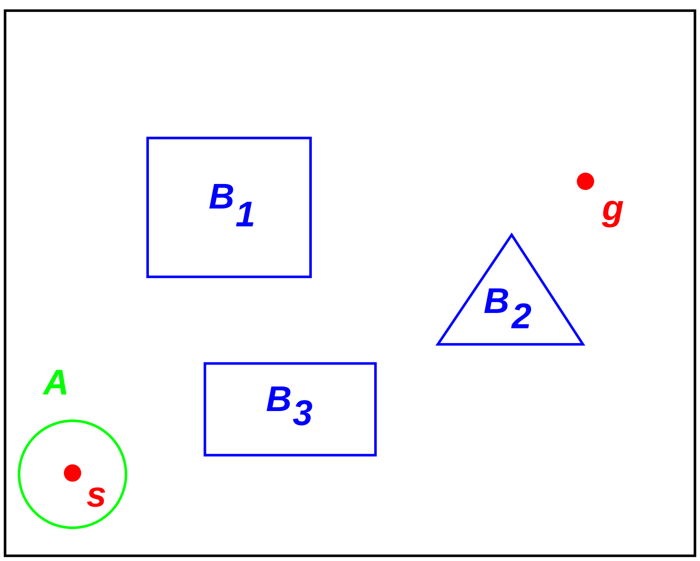
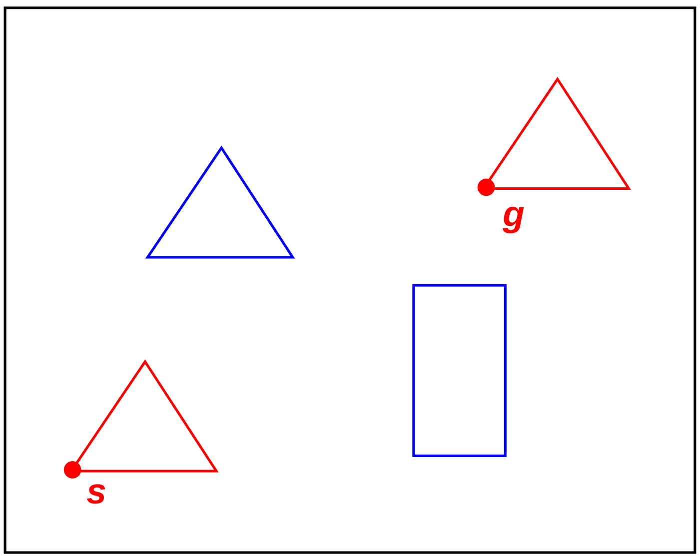
Gross Motion Planning

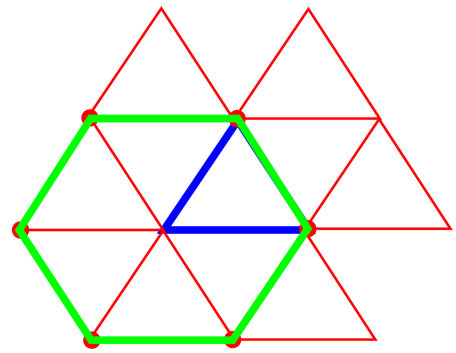
...given a moving object, A, initially in an unoccupied region of freespace, s, a set of stationary objects, B_i, at known locations, and a legal goal position, g,

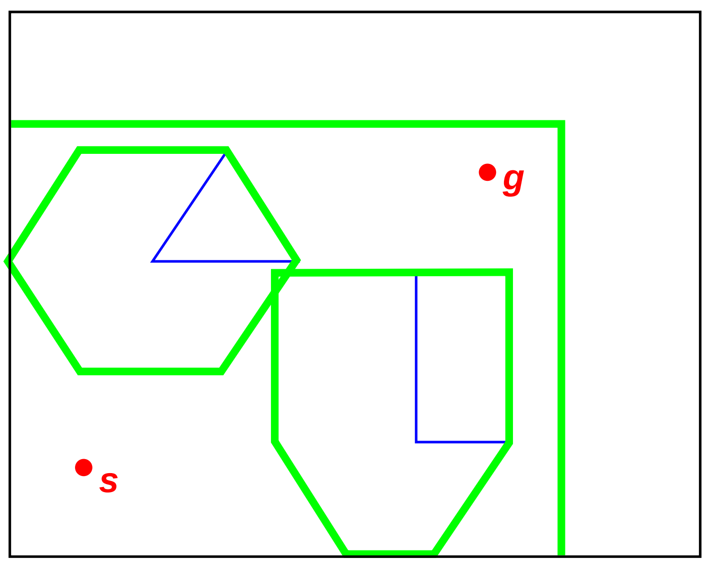
find a sequence of motions that take A from s to g without colliding with any obstacles, B_i.



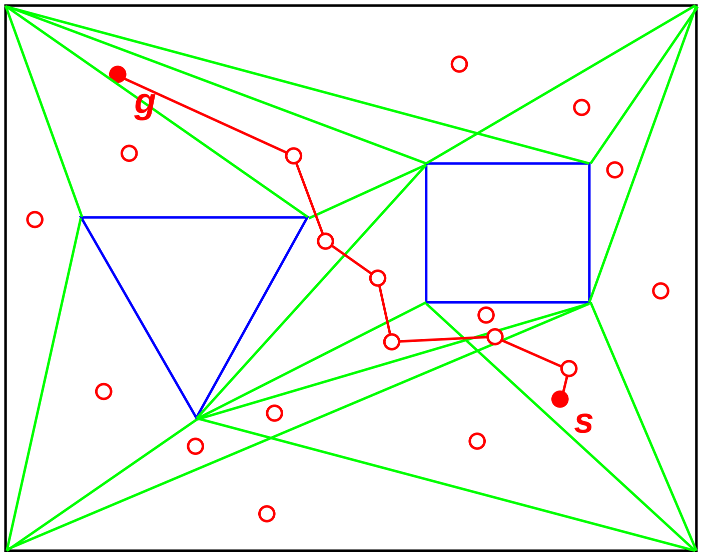
Path Planning Configuration Space







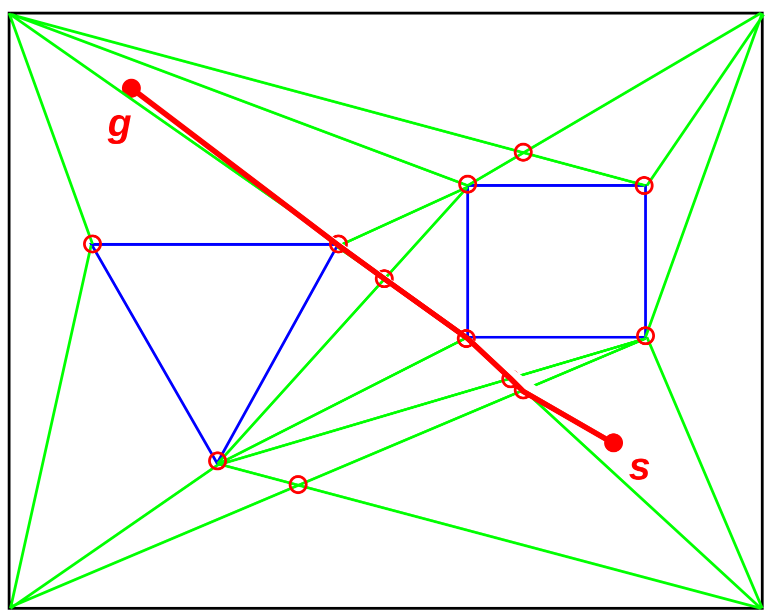
Path Planning Configuration Space



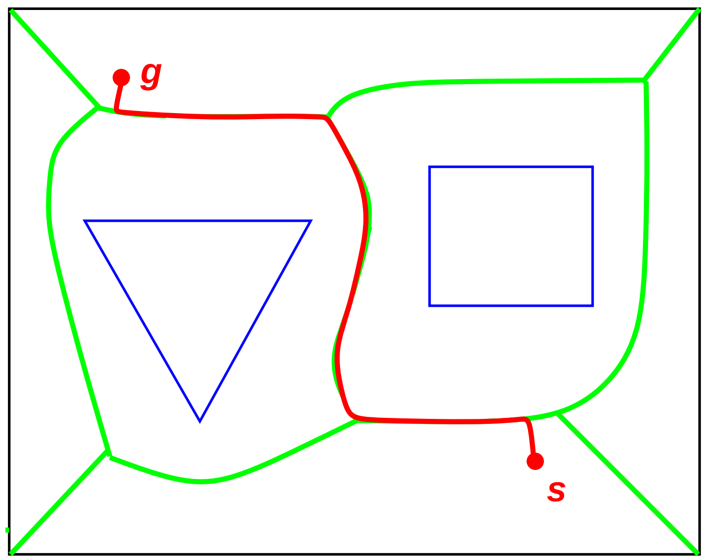
Simplicial Decomposition:
Schwartz and Sharir
Lozano-Perez
Canny

Vertex diagrams
exponential in number of environmental vertices
sensitive to geometric error/revision
unsmooth paths

Visibility Graphs



Path Planning Configuration Space

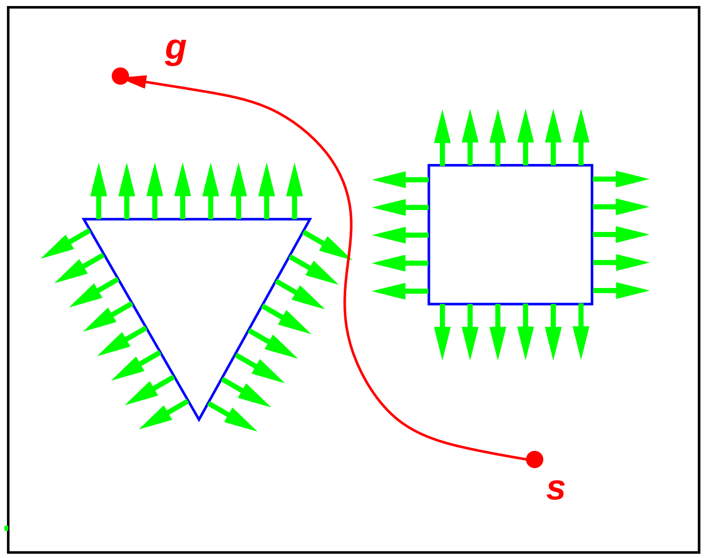


Voronoi diagram

Path Planning Configuration Space

Lozano-Perez - Mason - Taylor
generalized damper, configuration space, backward chaining, preimage

Path Planning Configuration Space

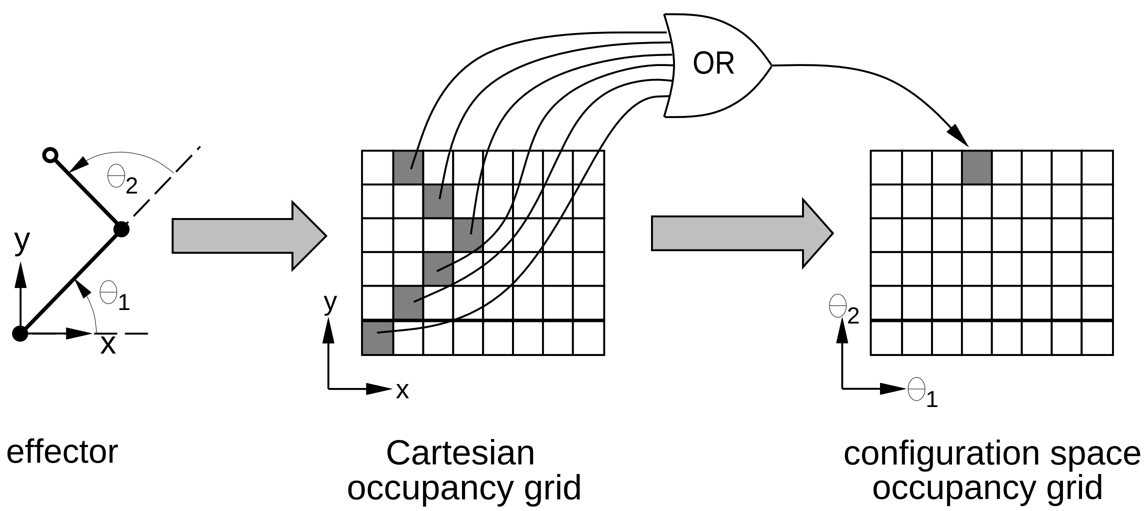


Potential Field Methods:
Khatib
Rimon and Koditschek
Arkin
Lyons

- obstacles are high potentials, goals are low potential
- robot behaves as a marble rolling down potential gradient
- physical analogs: gravitational, electrostatic
- smoothness
- analog implementations
- incremental/nonstationary models
- local minima

Manipulator Configuration Space

...navigation functions are designed in the configuration space of the manipulator...



Navigation Functions

define

$$V(\mathbf{x}, t)$$

a navigation function dependent on the state vector, $\mathbf{x}$, and time, t.

$$\frac{dV}{dt} = \frac{dV}{d\mathbf{x}} \frac{d\mathbf{x}}{dt}$$

where,

$$\frac{dV}{d\mathbf{x}} = \text{performance criteria -}$$

- obstacle avoidance
- perspective control
- grasp objective

$$\frac{d\mathbf{x}}{dt} = \text{system dynamics}$$

Navigation Functions

Analytic Diffeomorphisms — Rimon and Koditschek

- analytic — analytic functions can be represented by a series expansion and are continuously differentiable
- diffeomorphisms — a diffeomorphism is a differentiable map $f : S \mapsto S'$, which has a differentiable inverse $f^{-1} : S' \mapsto S$, therefore, f is continuous and differentiable.
- a diffeomorphism is called a conformal map if it is angle preserving (i.e., angles maps to the same angle, although lengths need not be preserved).
- transformations in *sphere* and *star* worlds which generate convergent control fields in the presence of obstacles.
- environmental complexity and system dimensionality are challenging practical issues.
- does not easily incorporate incremental observation

Admissible Control Functions

consider a candidate, multivariable control function, $f(q_0, q_1, ..., q_n)$, defined on domain, $\mathbf{Q}$,

the Hessian, $\partial^2 f / \partial \mathbf{Q}^2$, can be used to establish several important admissibility criteria...

$$\frac{\partial^2 f}{\partial \mathbf{Q}^2} = \begin{bmatrix} \frac{\partial^2 f}{\partial q_1^2} & \frac{\partial^2 f}{\partial q_1 \partial q_2} & \cdots & \frac{\partial^2 f}{\partial q_1 \partial q_n} \\ & \vdots & \vdots & \\ \frac{\partial^2 f}{\partial q_n \partial q_1} & \frac{\partial^2 f}{\partial q_n \partial q_2} & \cdots & \frac{\partial^2 f}{\partial q_n^2} \end{bmatrix}$$

Convex Control Functions: If the Hessian is positive semi-definite over the domain $\mathbf{Q}$, then the function f is convex over $\mathbf{Q}$.

Harmonic Control Functions: The harmonic constraint requires that the trace of the Hessian (or Laplacian) is identically zero:

$$\nabla^2 f = \frac{\partial^2 f}{\partial q_0^2} + \frac{\partial^2 f}{\partial q_1^2} + \cdots + \frac{\partial^2 f}{\partial q_n^2} = 0.$$

Sub-Harmonic Control Functions: This class of admissible control surfaces requires the Laplacian to be less than or equal to zero:

$$\nabla^2 f = \frac{\partial^2 f}{\partial q_0^2} + \frac{\partial^2 f}{\partial q_1^2} + \cdots + \frac{\partial^2 f}{\partial q_n^2} \leq 0.$$

Properties:

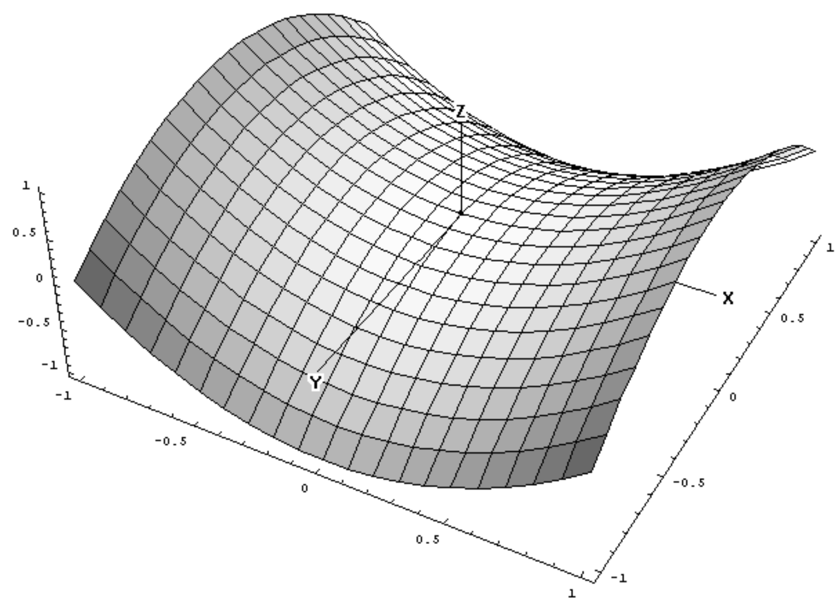
- complete up to model resolution
- closed under linear composition

Minima in Harmonic Potentials

consider n dimensions:

$$\sum_{i=1}^{n} \frac{\partial^2 \phi}{\partial q_i^2} = 0$$

if some i, if $\frac{\partial^2 \phi}{\partial q_i^2} > 0$ (concave upward), then there is another dimension, j, where $\frac{\partial^2 \phi}{\partial q_j^2} < 0$ to satisfy Laplace's constraint.



therefore, if you're not at a goal, there is always a way downhill...there are no local minima...

Other Properties

$$\nabla^2 V = \frac{d^2 V}{dx_1^2} + \ldots + \frac{d^2 V}{dx_n^2} = 0$$

Min-Max Property -

...in any compact neighborhood of freespace, the minimum and maximum of the function must occur on the boundary.

Mean-Value - up to truncation error, the value of the harmonic potential at a point in a lattice is the average of the values of its 2^n manhattan neighbors.

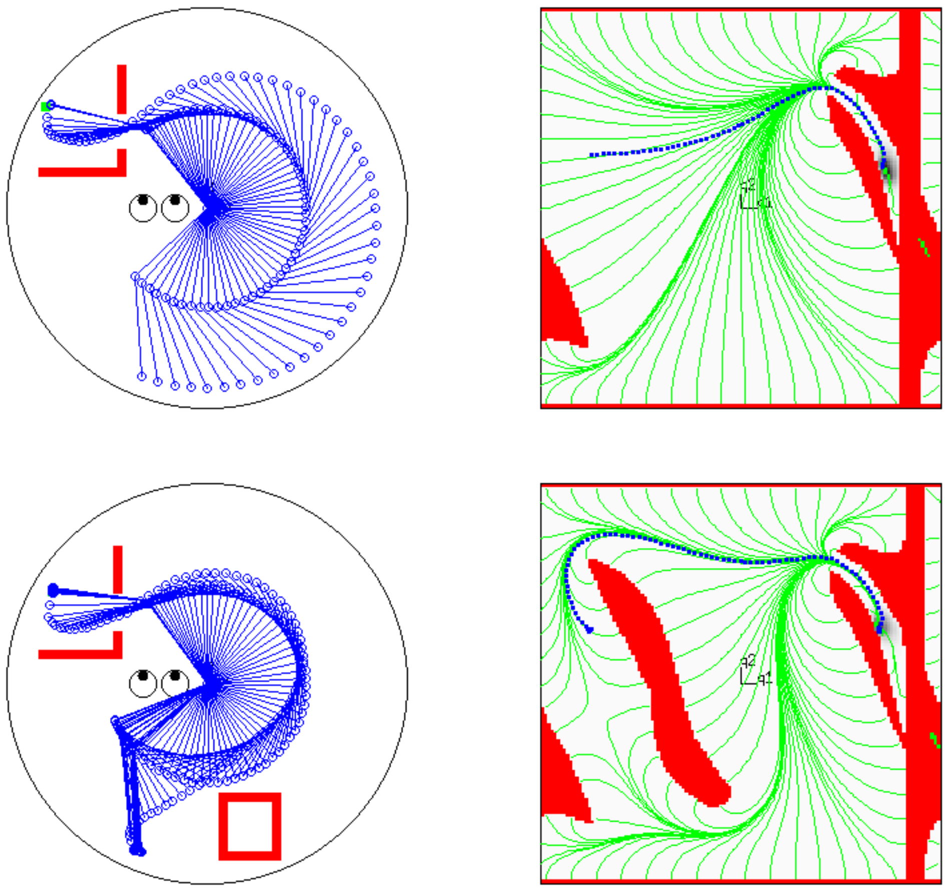
$$\begin{aligned} \nabla^2 \phi &\approx u(x_{i+1}, y_j) + u(x_{i-1}, y_j) + u(x_i, y_{j+1}) + u(x_i, y_{j-1}) - 4u(x_i, y_j)) \\ &= 0 \end{aligned}$$

Hitting Probabilities - if we denote $p(x)$ at state x as the probability that starting from x, a random walk process will reach an obstacle before it reaches a goal—$p(x)$ is known as the hitting probability

greedy descent on the harmonic function minimizes the hitting probability.

Harmonic Function Path Control

$$\nabla^2 V = \frac{d^2 V}{dx_1^2} + \ldots + \frac{d^2 V}{dx_n^2} = 0$$

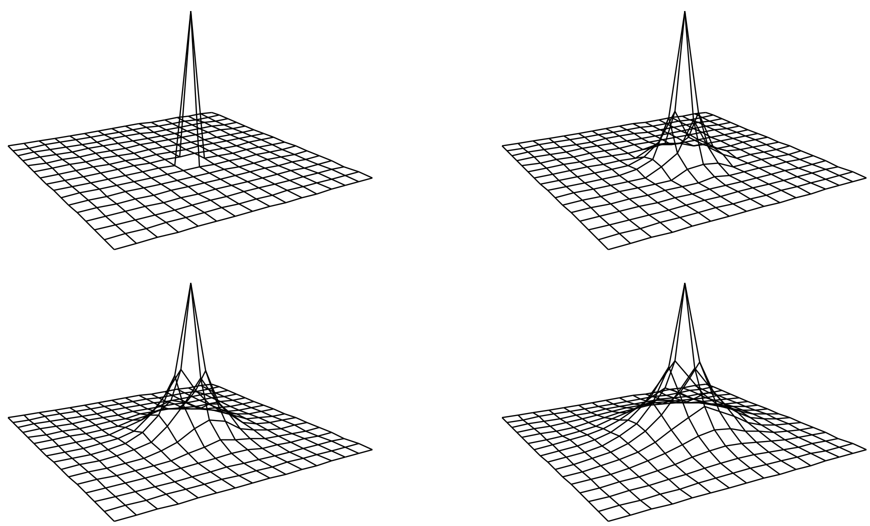


- obstacles fixed high potential, goals fixed low potential
- analogs: laminar fluid flow, steady state temperature distribution, electromagnetic fields
- computationally tractable — analog implementations

Harmonic Functions — numerical properties

Jacobi Iteration

$$u^{(k+1)}(x_i, y_j) = \frac{1}{4}\Big(u^{(k)}(x_{i+1}, y_j) + u^{(k)}(x_{i-1}, y_j) + u^{(k)}(x_i, y_{j+1}) + u^{(k)}(x_i, y_{j-1})\Big)$$



- SIMD (Single Instruction, Multiple Data) machine, a parallel numerical relaxation using one computational element per node in the discrete configuration space model
- relatively good convergence times for small problems on sequential machines.

Harmonic Functions — numerical properties

Gauss-Seidel

Iteration mixes values from different iterations:

$$\begin{aligned} u^{(k+1)}(x_i, y_j) \; &= \; \frac{1}{4}\Big(u^{(k)}(x_{i+1}, y_j) + u^{(k+1)}(x_{i-1}, y_j) + \\ &\qquad u^{(k)}(x_i, y_{j+1}) + u^{(k+1)}(x_i, y_{j-1})\Big) \end{aligned}$$

- In a single solution array replace each element by the average of its neighbors. Half of these neighbors are from iteration $k+1$, the other half are from iteration k.

Harmonic Functions — numerical properties

Successive Over Relaxation (SOR)

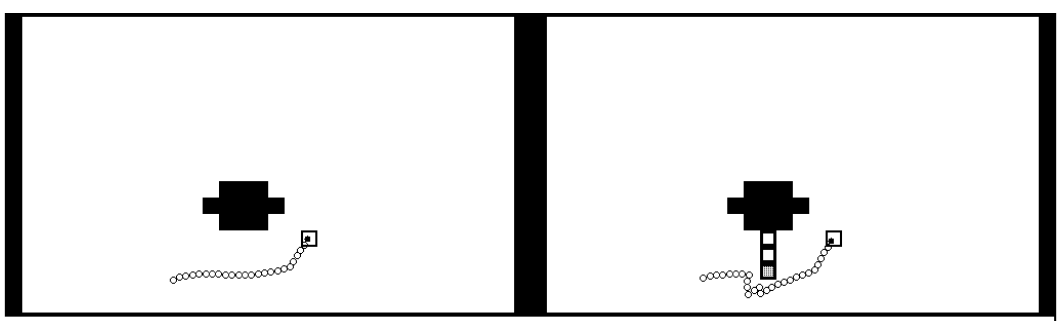
$$
\begin{aligned}
u^{(k+1)}(x_i, y_j) \;&=\; u^{(k)}(x_i, y_j) + \frac{\omega}{4}(u^{(k+1)}(x_{i+1}, y_j) + u^{(k+1)}(x_{i-1}, y_j) + \\
&\quad u^{(k)}(x_i, y_{j+1}) + u^{(k)}(x_i, y_{j-1}) - 4u^{(k)}(x_i, y_j))
\end{aligned}
$$

- k represents the iteration number,
- ω is the acceleration constant
- converges more rapidly than Gauss-Seidel or Jacobi iteration.

Reactive Potential Functions

Heuristic "Bump" Avoidance:

$$
\begin{aligned}
\dot{\vec{q}}_{EQ} &= -\left[\nabla\phi \times (\nabla\phi \times \vec{w})\right] \\
&= -\nabla\phi \times \begin{bmatrix} 0 & -\phi_{q_3} & \phi_{q_2} \\ \phi_{q_3} & 0 & -\phi_{q_1} \\ -\phi_{q_2} & \phi_{q_1} & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} \\
&= \begin{bmatrix} (\phi_{q_2}^2 + \phi_{q_3}^2) & -\phi_{q_1}\phi_{q_2} & -\phi_{q_1}\phi_{q_3} \\ -\phi_{q_1}\phi_{q_2} & (\phi_{q_1}^2 + \phi_{q_3}^2) & -\phi_{q_2}\phi_{q_3} \\ -\phi_{q_1}\phi_{q_3} & -\phi_{q_2}\phi_{q_3} & (\phi_{q_1}^2 + \phi_{q_2}^2) \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}
\end{aligned}
$$



Reactive Potential Function - Re-Modeling