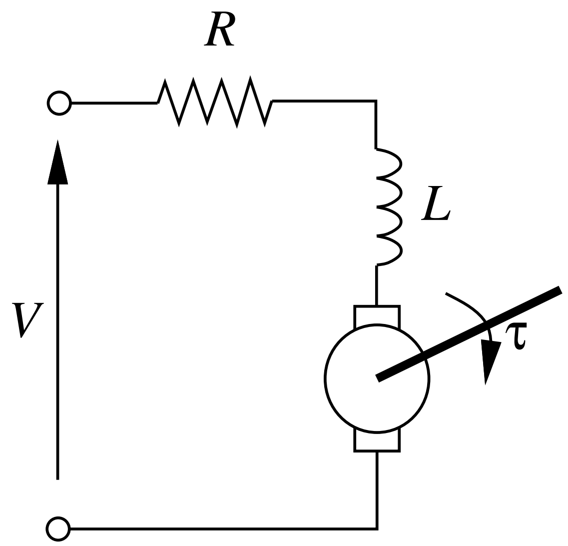
DC Motors - Equation of Motion



$$\sum \tau = J\ddot{\theta} = KI = K\left[\frac{V}{R} - \frac{K\dot{\theta}}{R}\right]$$

$$\ddot{\theta} + \frac{K^2}{JR}\dot{\theta} + \frac{KV}{JR} = 0$$

Laplace Transform

$$F(s) = \mathcal{L}[f(t)] = \int_0^\infty f(t)e^{-st}dt$$

where,

$$s = \sigma + j\omega$$

.

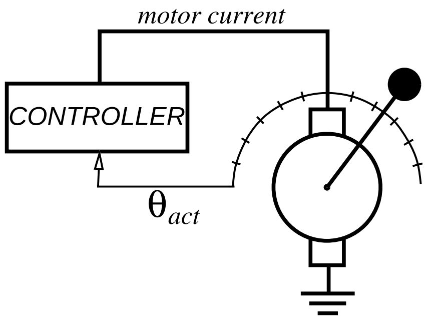
Name	f(t)	F(s)
unit impulse	$\delta(t)$	1
unit step	$u(t)$	$\frac{1}{s}$
ramp	t	$\frac{1}{s^2}$
exponential	e^{-at}	$\frac{1}{s+a}$
sine	$\sin at$	$\frac{a}{s^2+a^2}$
cosine	$\cos at$	$\frac{s}{s^2+a^2}$

Important Laplace Transform Theorems

$$F(s) = \mathcal{L}[f(t)] = \int_0^\infty f(t)e^{-st}dt$$

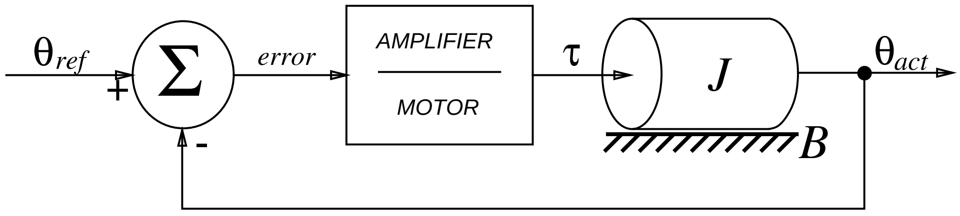
Name	Theorem
Derivative	$\mathcal{L}[\frac{df}{dt}] = sF(s) - f(0+)$
Integral	$\mathcal{L}\left[\int_0^t f(t)dt\right] = s^{-1}F(s)$
Shifting	$\mathcal{L}[f(t-t_0)u(t-t_0)] = e^{-t_0 s}F(s)$

EXAMPLE: DC Motor Electrodynamics

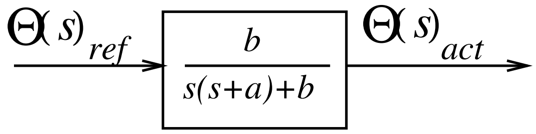


$$
\begin{aligned}
\tau &= A(\theta_{ref} - \theta_{act}) = Ae \\
&= J\ddot{\theta}_{act} + B\dot{\theta}_{act}
\end{aligned}
$$

$$
\begin{aligned}
0 &= \ddot{\theta}_{act} + \frac{B}{J}\dot{\theta}_{act} - \frac{A}{J}(\theta_{ref} - \theta_{act}) \\
0 &= (s^2 + as + b)\Theta_{act} - b\Theta_{ref}
\end{aligned}
$$



$$
\frac{\Theta(s)_{act}}{\Theta(s)_{ref}} = \frac{b}{s^2 + as + b}
$$



EXAMPLE: DC Motor Electrodynamics (cont)

Suppose that at $t = 0$, we apply a unit step angular reference

$$u(t) = 1 \qquad \Longrightarrow \qquad U(s) = \frac{1}{s}$$

if $b = (A/J) = 1$ and $a = (B/J) = 2$:

$$\Theta(s)_{act} = \frac{1}{s^2(s+2)+1} = \frac{1}{s(s+1)^2}$$

by partial-fraction expansion:

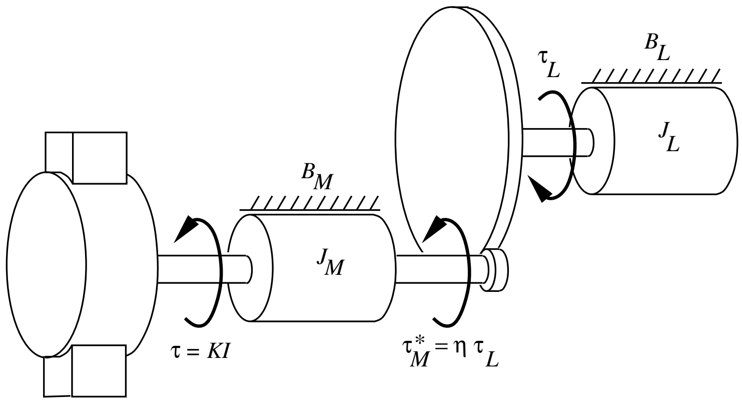
$$\begin{aligned}
\Theta(s)_{actt} &= \frac{1}{s(s+1)^2} \\
&= \frac{a}{s} + \frac{b}{(s+1)} + \frac{c}{(s+1)^2} \\
&= \frac{1}{s} + \frac{-1}{(s+1)} + \frac{-1}{(s+1)^2}
\end{aligned}$$

The inverse Laplace transform of these terms yields (from the tables):

$$\theta(t)_{act} = 1 - e^{-t} - te^{-t}$$

$\theta(0)_{act} = 0$, and $\theta(t \to \infty)_{act} = 1$

Actuators: DC Motors/Gearhead Combinations



if the transmission is perfectly efficient:

$$
\begin{aligned}
\tau_{out}\omega_{out} &= \tau_{in}\omega_{in} \\
\tau_{out}(\eta\omega_{in}) &= \tau_{in}\omega_{in} \\
\tau_{out} &= (1/\eta)\tau_{in}
\end{aligned}
$$

if $\eta = 0.01$, the output shaft carries one hundred times the torque at one hundredth the velocity of the input shaft

Actuators: DC Motors/Gearhead Combinations — Compound Loads

dynamic equation of motion

$$\tau = \left[J_M \ddot{\theta}_M + B_M \dot{\theta}_M\right] + \eta \left[J_L \ddot{\theta}_L + B_L \dot{\theta}_L\right]$$

but:

$$\begin{aligned} \theta_L &= \eta \theta_M, \\ \dot{\theta}_L &= \eta \dot{\theta}_M, \ and \\ \ddot{\theta}_L &= \eta \ddot{\theta}_M \end{aligned}$$

so:

$$\tau = \left[J_M + \eta^2 J_L\right] \ddot{\theta}_M + \left[B_M + \eta^2 B_L\right] \dot{\theta}_M$$

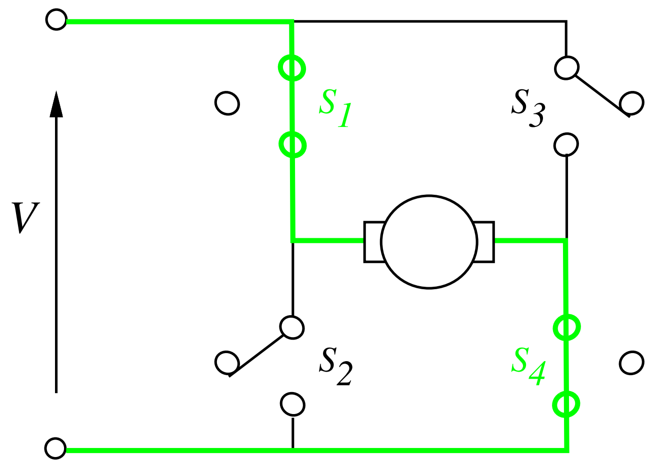
and:

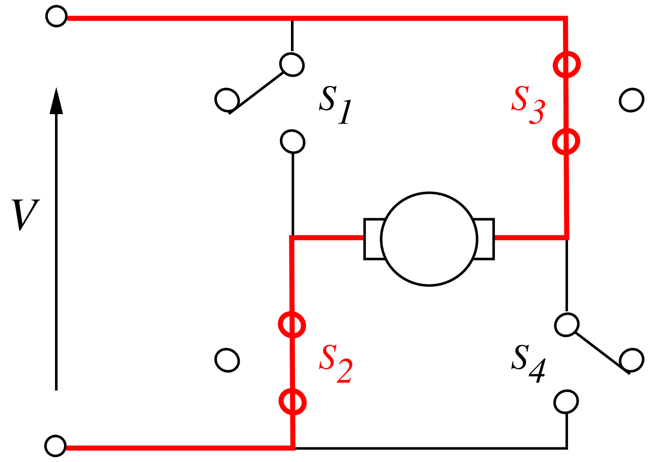
$$\begin{aligned} J_{eff} &= J_M + \eta^2 J_L \\ B_{eff} &= B_M + \eta^2 B_L \end{aligned}$$

More on Interfacing DC Motors

microprocessor should *control* the power supplied to the motor from an independent power source.

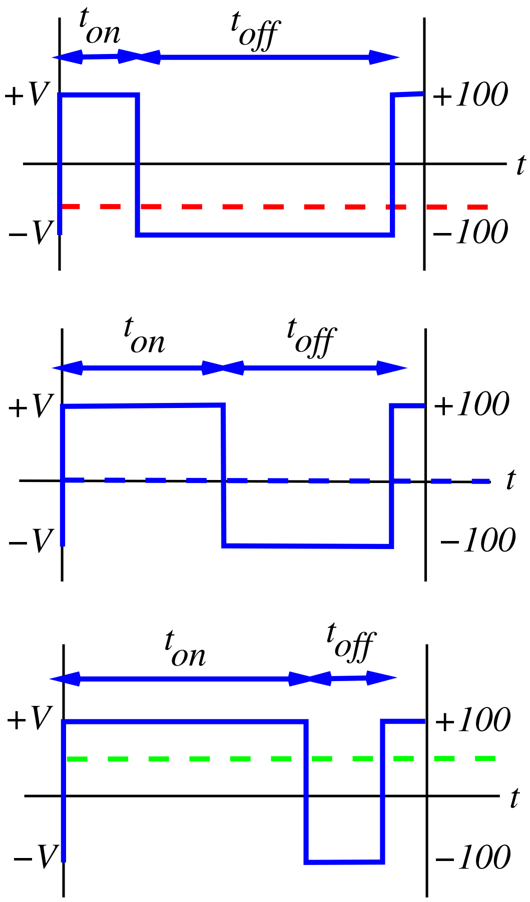
H-Bridge





Pulse Width Modulation

Suppose that S_1 and S_4 are closed (on) when S_2 and S_3 are open (off), and that all four switches toggle in unison...

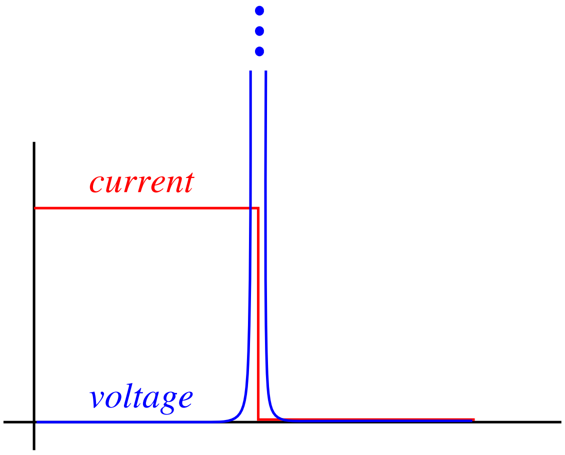


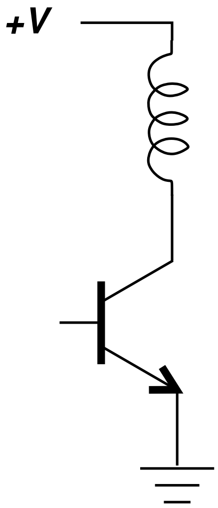
$$\text{Pulse Width Ratio} = \frac{t_{on}}{t_{on} + t_{off}}$$

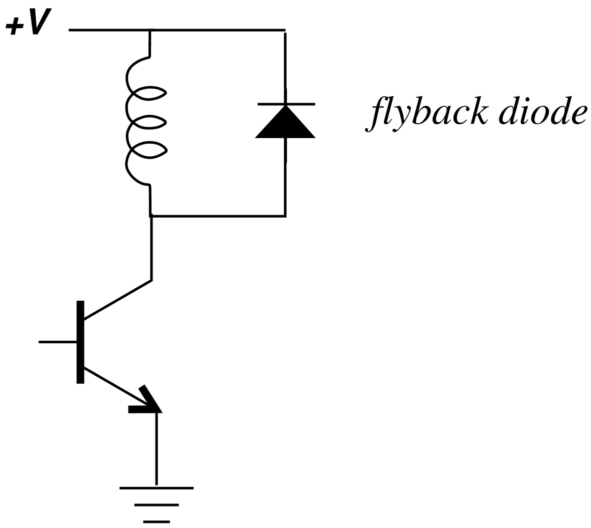
$$V_{motor} = -V + \text{PWR}(2V)$$

Switching Inductive Loads

$$V = L\frac{dI}{dt}$$







Bipolar H-Bridge - L293D