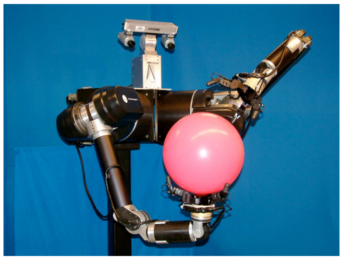
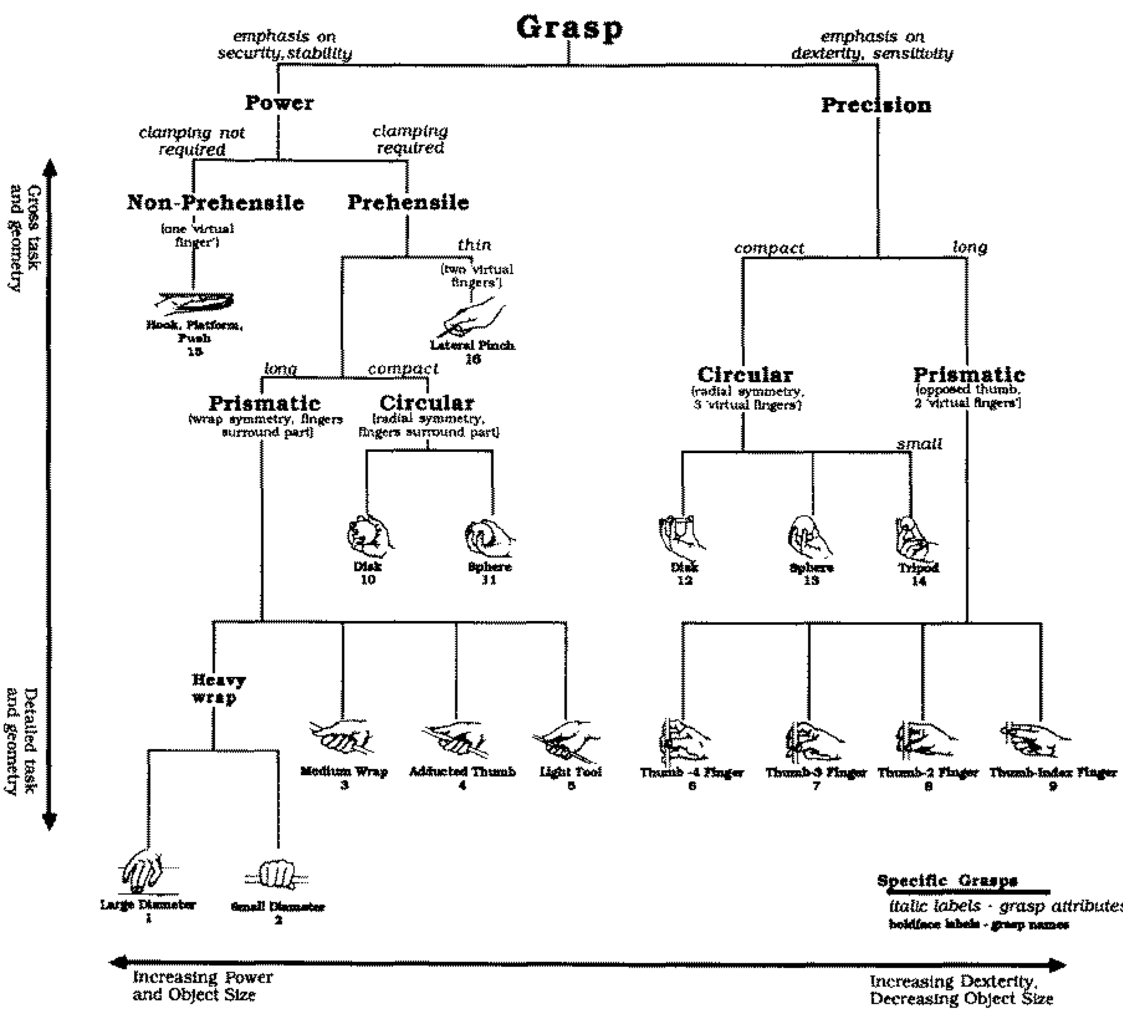
Grasping and Manipulation





Power vs. Precision — Connectivity vs. Mobility
Specificity vs. Generality

Mobility and Constraint in a Planar Mechanism

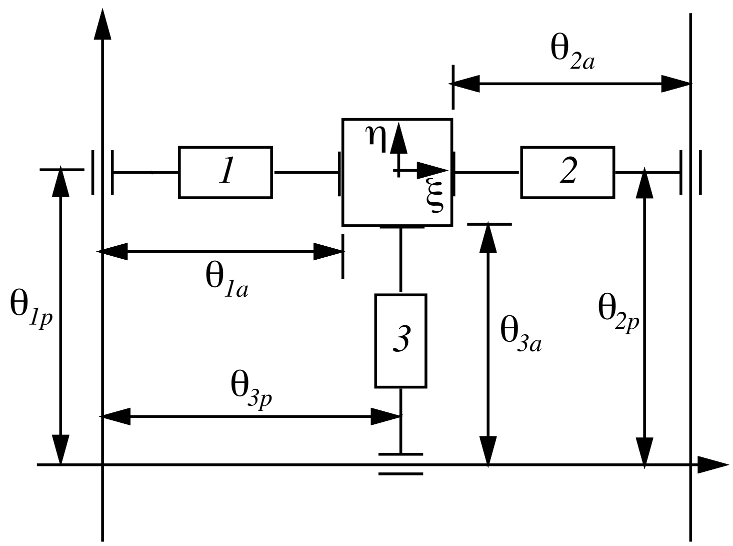
...if S represents differential displacements of the body in R^6 that are consistent with constraints imposed by fixed contacts, then the complement, $\overline{S}$, describes the set of forces that can be transmitted to the object by those contacts.

It must be true that $S \cup \overline{S} = R^6$ and $S \cap \overline{S} = \emptyset$.

To completely immobilize a body...

$$\begin{aligned} s_1 \cap s_2 \cap \cdots \cap s_n &= \emptyset, \text{ and} \\ \overline{s}_1 \cup \overline{s}_2 \cup \cdots \cup \overline{s}_n &= R^6, \end{aligned}$$

Mobility and Constraint in a Planar Mechanism



intersecting passive kinematics of each finger

A grasp immobilizes an object if the rank of S, $R(S) = 0$ and may be manipulated in the space spanned by $\overline{S}$.

ignoring unisense constraints for now...

finger #1: $\begin{bmatrix} \partial\xi \\ \partial\eta \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \partial\theta_{1a} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \partial\theta_{1p}$

finger #2: $\begin{bmatrix} \partial\xi \\ \partial\eta \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \partial\theta_{2a} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \partial\theta_{2p}$

finger #3: $\begin{bmatrix} \partial\xi \\ \partial\eta \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \partial\theta_{3a} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \partial\theta_{3p}$

Mobility and Constraint in a Planar Mechanism

considering just fingers 1 and 2...

$$S_{12} = \bigcap s_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \bigcap \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Therefore, fingers 1 and 2 allow unconstrained movement in one direction (the η direction).

considering fingers 1, 2, and 3...

$$S_{123} = \bigcap s_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \bigcap \begin{bmatrix} 0 \\ 1 \end{bmatrix} \bigcap \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \emptyset,$$

...when the three contacts are fixed in place by the actuators, and the object does not break contacts, then the object is fully constrained.

Furthermore, the wrench space of the contact configuration is the union of the motion constaints defined by the active degrees of freedom.

$$\overline{S}_{123} = \bigcup \overline{s}_i = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \bigcup \begin{bmatrix} -1 \\ 0 \end{bmatrix} \bigcup \begin{bmatrix} 0 \\ 1 \end{bmatrix} = R^2.$$

Screw **Nomenclature**

<u>*twist*</u>: differential displacements $\mathbf{t} \in R^6$

$$\boldsymbol{t} = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta\theta_x \\ \Delta\theta_y \\ \Delta\theta_z \end{bmatrix}$$

<u>*wrench*</u>: generalized force $\mathbf{w} \in R^6$

$$\boldsymbol{w} = \begin{bmatrix} f_x \\ f_y \\ f_z \\ m_x \\ m_y \\ m_z \end{bmatrix}$$

$\mathbf{t}$ and $\mathbf{w}$ do not constitute linear vector spaces!

work:

$$\mathbf{w}^T\mathbf{t} = [f_x \quad f_y \quad f_z \quad m_x \quad m_y \quad m_z] \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta\theta_x \\ \Delta\theta_y \\ \Delta\theta_z \end{bmatrix}$$

Form Closure

Reuleaux (1876) - "the condition under which a positive combination of contact wrenches derived from frictionless contacts can resist perturbation forces."

form closure can be defined solely in terms of mobility without specifying contact forces at all

Definition 0.0.1 (Form Closure) - *a condition of complete restraint in which any object twist* $\in R^6$ *is inconsistent with rigid body assumption for objects and fixturing elements.*

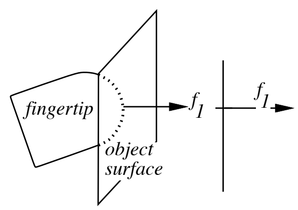
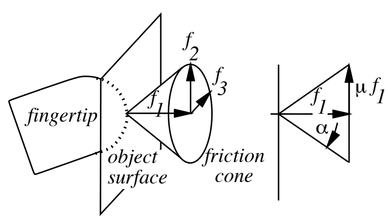
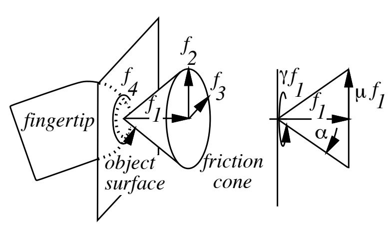
- Reuleaux
  - planar bodies require at least four frictionless contacts for form closure in R^3, and
  - *exceptional* surfaces exist for which form closure is impossible given any number of frictionless point contacts.
- Somoff (1897) proved that at least 7 frictionless point contacts are necessary for form closure in R^6
- Mishra, Schwartz and Sharir (1987) - established an upper bound of 6 frictionless point contacts on planar objects with piecewise smooth contours, and 12 for the spatial case (except for Reuleaux's exceptional surfaces).

Force Closure

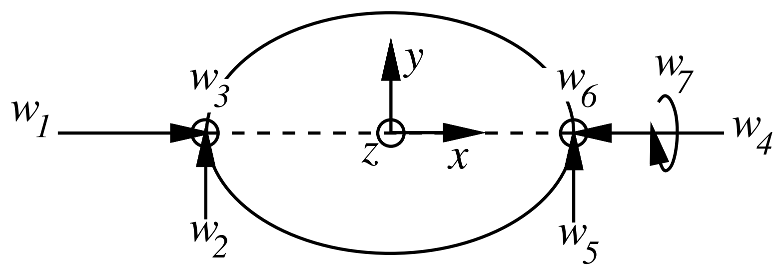
1. The grasped object is in quasistatic equilibrium, there are no net forces or moments,
2. all forces are applied within the cone of friction so that there is no slippage, and,
3. an externally applied force can be resisted by finger forces with a finite and controllable deflection.

Definition 0.0.2 (Force Closure) - *a grasp configuration is force closure if the grasp configuration is in equilibrium for any arbitrary external load.*

Contact Types

contact type	geometry	wrench basis	constraints
frictionless point contact		$\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$	$\mathbf{f}_1^T \mathbf{f}_1 \geq 0$
point contact with friction		$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\mathbf{f}_2^T \mathbf{f}_2 + \mathbf{f}_3^T \mathbf{f}_3 \leq \mu(\mathbf{f}_1^T \mathbf{f}_1)$ $\mathbf{f}_1^T \mathbf{f}_1 \geq 0$
soft finger		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\mathbf{f}_2^T \mathbf{f}_2 + \mathbf{f}_3^T \mathbf{f}_3 \leq \mu(\mathbf{f}_1^T \mathbf{f}_1)$ $\mathbf{f}_4^T \mathbf{f}_4 \leq \gamma(\mathbf{f}_1^T \mathbf{f}_1)$ $\mathbf{f}_1^T \mathbf{f}_1 \geq 0$

Grip Jacobian



assume that unit contact forces, $\mathbf{f}_i \in R^3$, are independent—that is, we assume that frictional forces are independent of normal forces.

- left contact ($\mathbf{f}_1$, $\mathbf{f}_2$, and $\mathbf{f}_3$) is a point contact with friction
- right contact ($\mathbf{f}_4$, $\mathbf{f}_5$, $\mathbf{f}_6$, and $\mathbf{f}_7$) corresponds to a soft-finger contact.

the $\mathbf{f}_i$ create forces and moments, $\boldsymbol{w}_i = [\mathbf{f}_i \;\; \mathbf{r}_i \times \mathbf{f}_i]^T$, in the body frame.

Grip Jacobian - n column vectors describing the body frame wrenches corresponding to each of the n contact forces.

$$\mathcal{W} = \begin{array}{c} \begin{matrix} \mathbf{w}_1 & \mathbf{w}_2 & \mathbf{w}_3 & \mathbf{w}_4 & \mathbf{w}_5 & \mathbf{w}_6 & \mathbf{w}_7 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Grip Jacobian

reference wrench, $\mathbf{w}$, can be written as a linear combination of contact wrenches if a solution for $\boldsymbol{\lambda} = [\lambda_1 \lambda_2 \cdots \lambda_7]^T$ satisfies

$$\mathbf{w} = \mathcal{W}\boldsymbol{\lambda}.$$

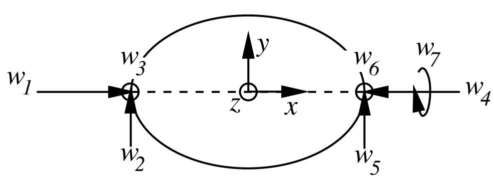
where solutions $\boldsymbol{\lambda}$ have homogeneous and particular parts,

$$\boldsymbol{\lambda} = \boldsymbol{\lambda}_p + \boldsymbol{\kappa}^T \boldsymbol{\lambda}_h$$

$\boldsymbol{\lambda}_h$ is the **homogeneous part** of the solution and describes combinations of contact forces that impart zero net force to the object.

$$\mathcal{W}\boldsymbol{\lambda}_h = 0$$

- $\mathcal{W}$ must be full rank to achieve arbitrary reference wrenches
- $\boldsymbol{\lambda}$ must satisfy inequality constraints for unisense normal forces and contact friction.



$$\boldsymbol{\lambda}_h = [1\ 0\ 0\ 1\ 0\ 0\ 0]^T$$

...the existence of a null space in the grip Jacobian with rank at least 1 is a necessary condition for force closure.

Grasp Planning

1. Salisbury (1982) - analytical framework for evaluating grasp stability in which the stiffness matrix that characterizes a grasp must be positive definite.
2. Cutkosky (1985) - grasp stability depends on force distributions and local curvature
3. Montana (1988) - contact grasp stability evaluates the ability of a perturbed grasp geometry to return to an equilibrium configuration—kinematic description of rolling contacts
4. Nguyen (1989) - all force closure grasps are *stabilizable* by actively modulating contact forces.
5. Hemami (ca. 1989) - treated dynamic stability using the methods of Lyapunov.
6. Ferrari (1992) - grasp metrics for judging the quality of a grasp for planning methods.
7. Bicchi (1995) - tested force closure and a quality metric for a grasp given friction, contact forces, and constraints on applied forces.

however, the execution of a grasp is notoriously sensitive to the value of the friction coefficient, rolling and sliding contacts, spatial precision, and kinematic limitations—a feedback basis for grasp execution is required