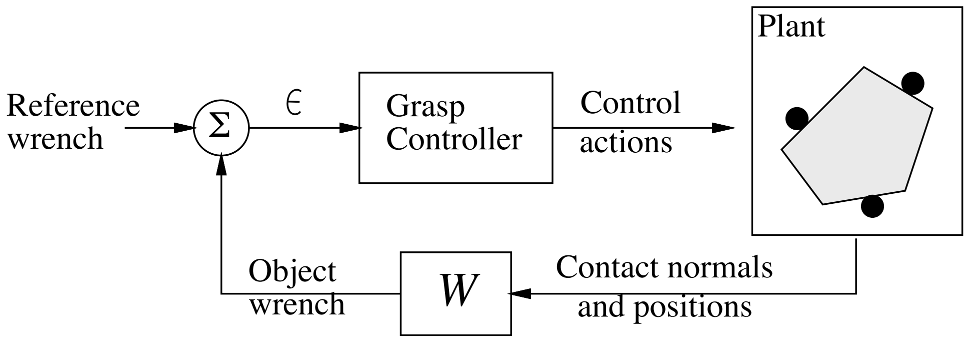
Closed-Loop, Sensor-Driven Grasp Control



minimize the squared wrench residual ϵ:

$$\begin{aligned}\epsilon &= \boldsymbol{\rho}^T \mathbf{M}\, \boldsymbol{\rho} \\ &= \left(\frac{1}{k}\sum_{i=1}^{k} \boldsymbol{\omega}_i\right)^T \mathbf{M} \left(\frac{1}{k}\sum_{i=1}^{k} \boldsymbol{\omega}_i\right)\end{aligned}$$

The squared residual, ϵ, is minimized by following the negative gradient of ϵ with respect to the contact configuration variable, $\mathbf{q}$.

Grasp Control

differential grasping actions are derived from $-\nabla_q \epsilon$,

$$
\begin{aligned}
\frac{\partial \epsilon}{\partial q_j} &= 2 \left(\frac{1}{k} \sum_i \boldsymbol{\omega}_i \right)^T \frac{\partial \left(\frac{1}{k} \sum_i \boldsymbol{\omega}_i \right)}{\partial q_j} \\
&= 2 \left(\frac{1}{k} \sum_i \boldsymbol{\omega}_i \right)^T \frac{\partial \boldsymbol{\omega}_j}{\partial q_j} \frac{1}{k} \\
\frac{\partial \epsilon}{\partial q_j}_{(1\times1)} &\propto \boldsymbol{\rho}^T_{(1\times6)} \left(\frac{\partial \boldsymbol{\omega}_j}{\partial q_j} \right)_{(6\times1)}
\end{aligned}
$$

where $\frac{\partial \boldsymbol{\omega}_j}{\partial q_j}$ is the partial derivative of the j^{th} contact wrench with respect to the coordinate q_j.

In vector notation

$$
\frac{\partial \epsilon}{\partial \mathbf{q}} \propto \boldsymbol{\rho}^T \left[\frac{\partial \boldsymbol{\omega}_1}{\partial \mathbf{q}_1} \;\; \frac{\partial \boldsymbol{\omega}_2}{\partial \mathbf{q}_2} \;\; \cdots \;\; \frac{\partial \boldsymbol{\omega}_n}{\partial \mathbf{q}_n} \right]_{6Xn} = \boldsymbol{\rho}^T \frac{\partial \mathcal{W}}{\partial \mathbf{q}}
$$

Grasp Control

minima in $\boldsymbol{\rho}^T \frac{\partial \mathcal{W}}{\partial \mathbf{q}}$ are candidate grasp configurations with non-zero $R(\mathcal{W})$

- where $\boldsymbol{\rho} = 0$ - adequate friction in the respective contacts can lead to full rank in $\mathcal{W}$ with at least one homogeneous solution, λ_h—such a grasp is force closure and, therefore, statically stabilizable.
- where $\boldsymbol{\rho} \neq 0$ and $\boldsymbol{\rho}^T \frac{\partial \mathcal{W}}{\partial \mathbf{q}} = 0$, some of equilibrium grasp configurations can be stabilized by contact force modulation or by frictional forces, others are truly local minima and must be avoided.

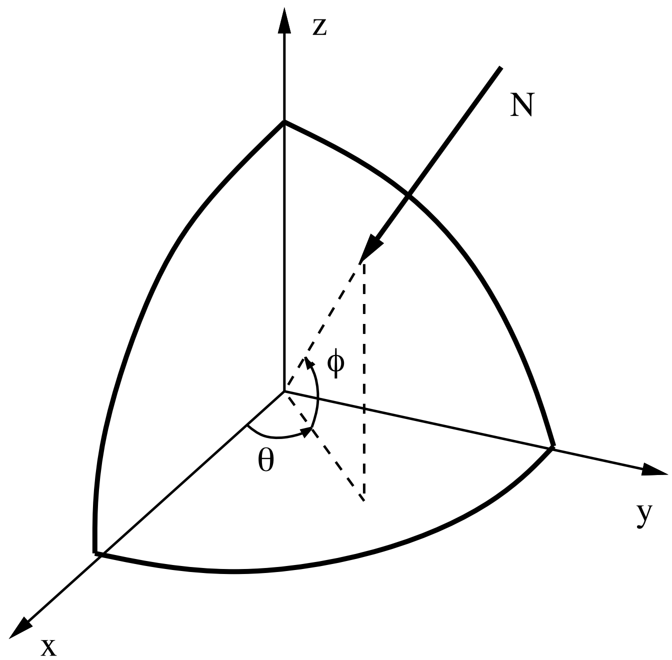
$\frac{\partial \mathcal{W}}{\partial \mathbf{q}}$ and $\boldsymbol{\rho}$ require an expression for the contact wrench as a function of contact coordinates. This function depends directly on the geometry of the object. To build controllers applicable to a range of object geometries, two local surfaces types are considered.

- **force residual control** is derived from a model of contact wrenches derived from frictionless point contacts on a sphere
- **moment residual control** is derived from a model of frictionless point contacts on an infinite plane defined by the contact normal

...these models yield artificial potentials with unique minima and represent two local surface hypotheses that serve as a constructive basis for approximating a range of object geometries

Residual Force Control

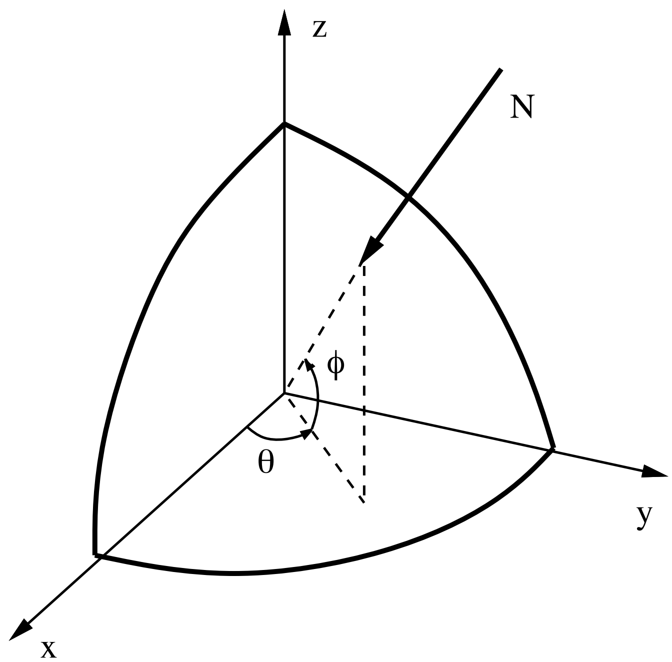
consider a model describing how object frame contact wrenches vary with contact coordinate (θ and ϕ) on the surface of a sphere



the corresponding object frame wrench, $\mathbf{w}(\theta, \phi)$, can be written as:

$$\mathbf{w}(\theta, \phi) \;=\; \begin{bmatrix} w_{fx} \\ w_{fy} \\ w_{fz} \\ w_{mx} \\ w_{my} \\ w_{mz} \end{bmatrix} = \begin{bmatrix} -\cos(\theta)\cos(\phi) \\ -\sin(\theta)\cos(\phi) \\ -\sin(\phi) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Residual Force Control - cont.



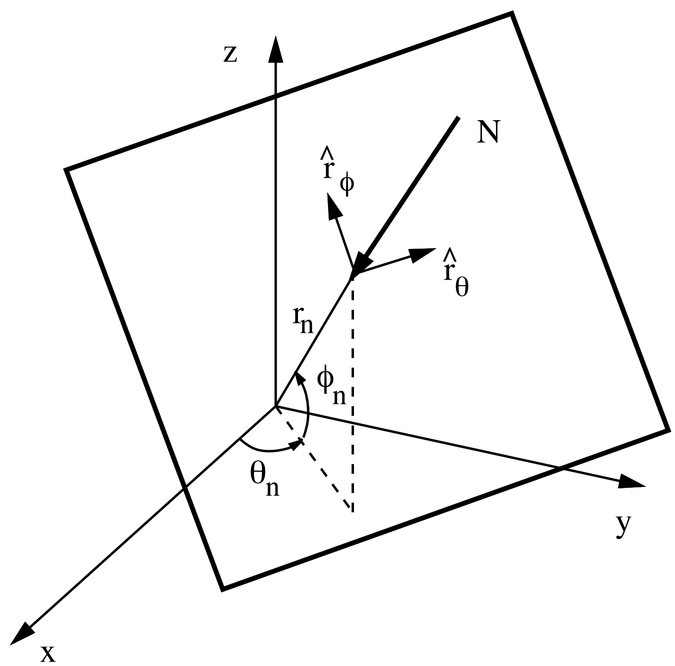
and the change in the contact wrench due to a differential displacement of the contact is

$$\begin{bmatrix} w_{\theta_i} & w_{\phi_i} \end{bmatrix}_f = \begin{bmatrix} \frac{\partial \mathbf{w}}{\partial \theta_i} & \frac{\partial \mathbf{w}}{\partial \phi_i} \end{bmatrix} = \begin{bmatrix} \sin(\theta_i)\cos(\phi_i) & \cos(\theta_i)\sin(\phi_i) \\ -\cos(\theta_i)\cos(\phi_i) & \sin(\theta_i)\sin(\phi_i) \\ 0 & -\cos(\phi_i) \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

the force residual potential is unimodal and gradient descent on this wrench model considers forces exclusively

Residual Moment Control

minimize the moment residual by controlling the position of the contact (r_θ, r_ϕ) on a locally planar surface defined by parameters (r_0, θ_0, ϕ_0).



contact moments, $\mathbf{m} = \mathbf{r} \times \mathbf{f}$, vary linearly in the plane, passing through zero when $r_\theta = 0$ and $r_\phi = 0$. The resulting wrench model is:

$$\mathbf{w}(\theta, \phi) = \begin{bmatrix} w_{fx} \\ w_{fy} \\ w_{fz} \\ w_{mx} \\ w_{my} \\ w_{mz} \end{bmatrix} = \begin{bmatrix} -\cos(\theta_0)\cos(\phi_0) \\ -\sin(\theta_0)\cos(\phi_0) \\ -\sin(\phi_0) \\ -r_\theta \sin(\phi_0)\cos(\theta_0) + r_\phi \sin(\theta_0) \\ -r_\theta \sin(\phi_0)\sin(\theta_0) - r_\phi \cos(\theta_0) \\ r_\theta \cos(\phi_0) \end{bmatrix}.$$

Residual Moment Control

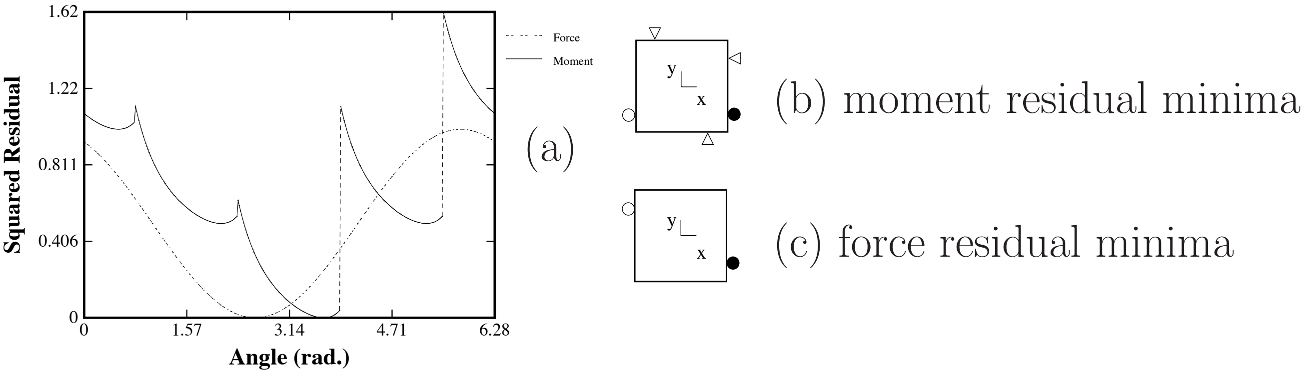
This model yields correct instantaneous wrenches (and therefore wrench residuals) for all objects provided that (r_0, θ_0, ϕ_0) are known or measurable.

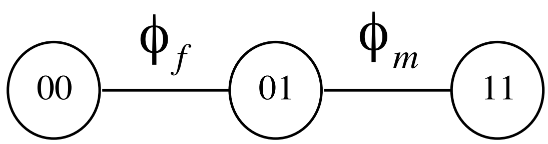
moment residual control assumes that these parameters are constant in the neighborhood of contact i, so that

$$\begin{bmatrix} w_{r_{\theta_i}} & w_{r_{\phi_i}} \end{bmatrix}_m = \begin{bmatrix} \frac{\partial \mathbf{w}}{\partial r_{\theta_i}} & \frac{\partial \mathbf{w}}{\partial r_{\phi_i}} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -\cos(\theta_{0i})\sin(\phi_{0i}) & \sin(\theta_{0i}) \\ -\sin(\theta_{0i})\sin(\phi_{0i}) & -\cos(\theta_{0i}) \\ \cos(\phi_{0i}) & 0 \end{bmatrix}$$

the moment residual potential is unimodal and gradient descent on this wrench model considers moments exclusively

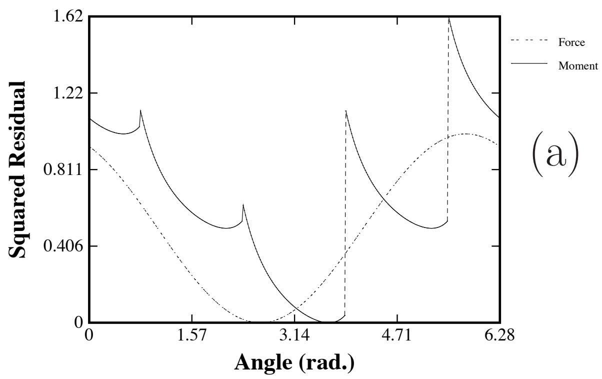
Control Composition - Coelho 2001



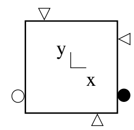


Theorem 0.0.1 (Regular Convex Polytopes) *...globally optimal grasp configuration is achieved by first performing gradient descent on the residual force potential and then following the gradient of the residual moment potential.*

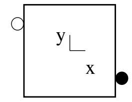
Control Composition - Platt 2005



(a)

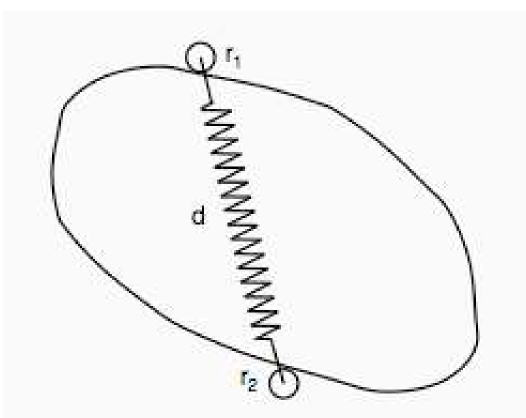


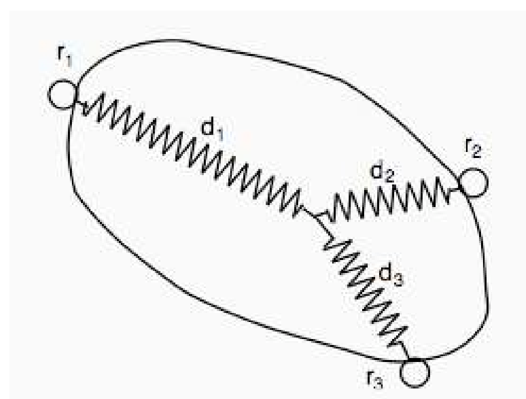
(b) moment residual minima



(c) force residual minima

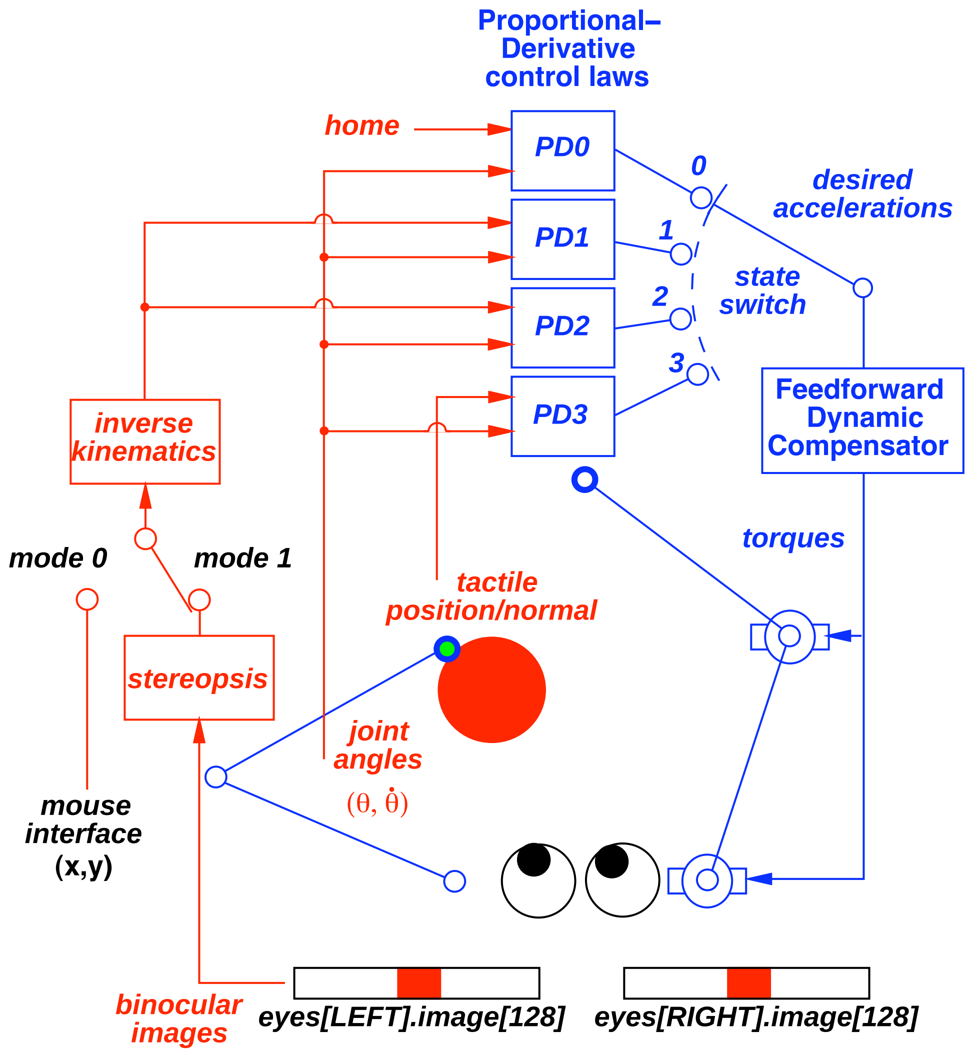
$$\phi_m \lhd \phi_f$$





Theorem 0.0.2 (Convex Objects) *$\phi_m \lhd \phi_f$ is stable in the sense of Lyapunov to globally optimal grasp configurations.*

Grasp Control Project - Control Structure



Grasp Control Project - Actions

PD0 - the PD controller with arm home position as goal.

PD1 - The PD controller with goals placed at a slight radial offset from the visual edges, so that the arms are near, but not yet touching, the ball.

PD2 - a "gentle" PD control that reaches to the centroid of the ball. This controller terminates when tactile_probe() returns TRUE (or the contact force reaches a threshold), and

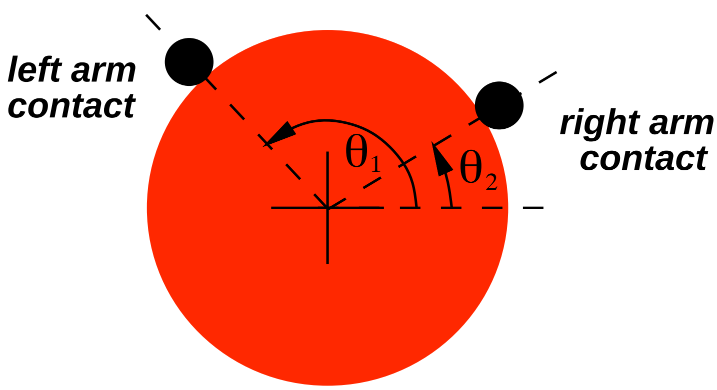
PD3 - a PD controller to a goal that is the relative angular displacement from the current location (and small radial offset) determined by the force residual controller. This new location serves to initialize a new tactile probe by re-initializing PD2.

Grasp Control Project - Wrench Closure Controller

generalized wrench,

$$\mathbf{w}_i^T = [f_x \ \ f_y \ \ m_z] = [-cos(\theta_i) \ \ -sin(\theta_i) \ \ 0]$$

is a function solely of θ. Moreover, he has only two contacts, so we can simplify the expression for the gradient of the force residual.



Grasp Control Project - Procedure

1. Write a scalar expression for the squared residual $\epsilon = \rho^T \rho$. You will compare this scalar to a suitable threshold to determine if your grasp controller has converged.

2. Write the Jacobian for the force residual, $J^F = [\frac{\partial \epsilon}{\partial \theta_1} \quad \frac{\partial \epsilon}{\partial \theta_2}]_{(1\times 2)}$ so that, $\Delta\epsilon_{(1\times 1)} = J^F_{(1\times 2)} \Delta\boldsymbol{\theta}_{(2\times 1)}$.

3. The pseudoinverse of J^F is written

$$(J^F)^{\#}_{(2\times 1)} = (J^F)^T_{(2\times 1)} \left[J^F_{(1\times 2)} (J^F)^T_{(2\times 1)} \right]^{-1}$$

. Derive the closed form solution for $(J^F)^{\#}$.

4. PD3 must compute a new reference position for an arm movement that eliminates the remaining force residual, i.e it prescribes and angular displacement of $\Delta\boldsymbol{\theta}_{(2\times 1)} = J^{\#}_{(2\times 1)} \Delta\epsilon_{(1\times 1)} = J^{\#}(-\epsilon)$ and perhaps a small radial offset as well, so the arm doesn't disturb the ball too much while delicately probing the ball. Using your result from parts 1 and 3, compute a new Cartesian goal for PD3.

Grasp Control Project - Procedure

The logic that controls the switching sequence is called the REACH-GRASP schema. It determines the sequence of control tasks that together comprise the integrated reach and grasp behavior.