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Grasping

- mobility and connectivity analysis
- form closure
- the grasp Jacobian
- force closure

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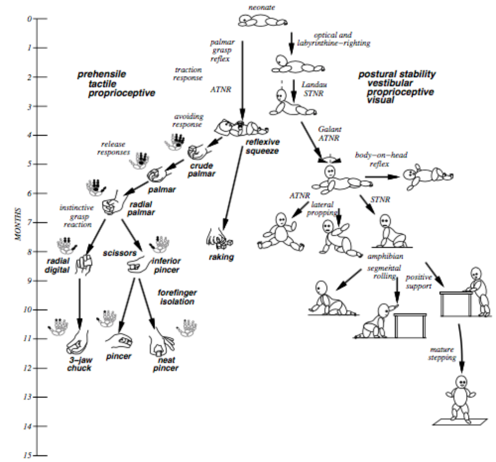
Grasping and Manipulation



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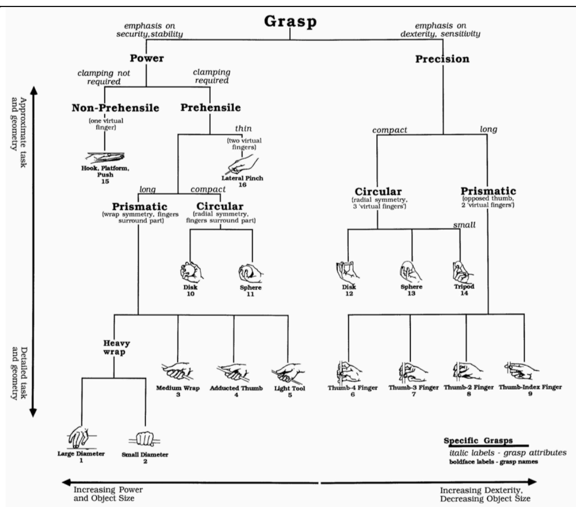
Grasping and Manipulation



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Screw Nomenclature: Wrenches and Twists

<u>*twist*</u>: generalized velocity

$$\mathbf{v} = \begin{bmatrix} v_x \\ v_y \\ v_z \\ \omega_x \\ \omega_y \\ \ldots \end{bmatrix}$$

<u>*wrench*</u>: generalized force

$$\mathbf{w} = \begin{bmatrix} f_x \\ f_y \\ f_z \\ m_x \\ m_y \\ m_z \end{bmatrix}$$

<u>*power*</u>: $\mathbf{w}^T\mathbf{v} = [f_x \;\; f_y \;\; f_z \;\; m_x \;\; m_y \;\; m_z] \begin{bmatrix} v_x \\ v_y \\ v_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$

v and **w** do not constitute linear vector spaces!

Grasp Analysis - Mobility and Connectivity

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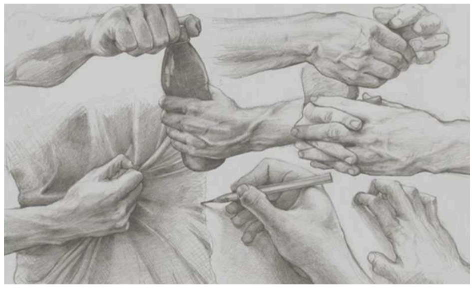

$\mathbf{v} \in V$: object twists consistent with contact constraints; and
$\overline{\mathbf{v}} \in \overline{V}$: object twists that are restricted by contact constraints.

$$span\{V \cup \overline{V}\} = \mathbb{R}^6 \qquad and \qquad \{V \cap \overline{V}\} = \{\emptyset\}$$

Grasp Analysis - Mobility and Connectivity

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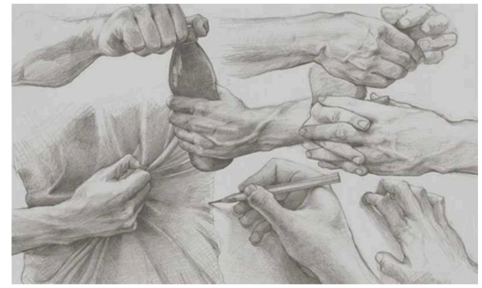

for a system of n contacts to immobilize a body:

$$\{\mathbf{v}_1 \cap \mathbf{v}_2 \cap \cdots \cap \mathbf{v}_n\} = \{\emptyset\}, \quad \text{and}$$

$$span\{\overline{\mathbf{v}}_1 \cup \overline{\mathbf{v}}_2 \cup \cdots \cup \overline{\mathbf{v}}_n\} = \mathbb{R}^6$$

Grasp Analysis - Mobility and Connectivity

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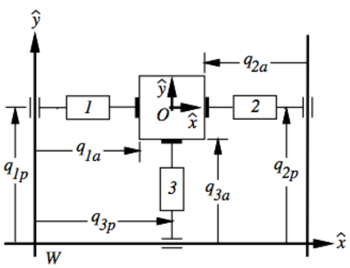

"velcro" contacts

imagine that active dof can move
to a position and then "lock" in place

finger #1:
$$\begin{bmatrix} v_x \\ v_y \end{bmatrix}_O = \begin{bmatrix} 1 \\ 0 \end{bmatrix}_O \dot{q}_{1a} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}_O \dot{q}_{1p} = \overline{\mathbf{v}}_1 \quad \dot{q}_{1a} + \mathbf{v}_1 \quad \dot{q}_{1p}.$$

finger #2:
$$\begin{bmatrix} v_x \\ v_y \end{bmatrix}_O = \begin{bmatrix} -1 \\ 0 \end{bmatrix}_O \dot{q}_{2a} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}_O \dot{q}_{2p} = \overline{\mathbf{v}}_2 \quad \dot{q}_{2a} + \mathbf{v}_2 \quad \dot{q}_{2p},$$

finger #3:
$$\begin{bmatrix} v_x \\ v_y \end{bmatrix}_O = \begin{bmatrix} 0 \\ 1 \end{bmatrix}_O \dot{q}_{3a} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}_O \dot{q}_{3p} = \overline{\mathbf{v}}_3 \quad \dot{q}_{3a} + \mathbf{v}_3 \quad \dot{q}_{3p}.$$

Grasp Analysis - Mobility and Connectivity

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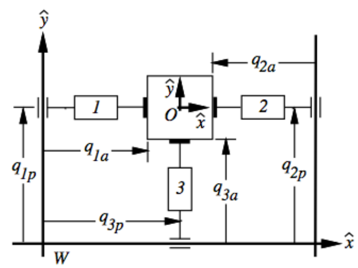

finger #1:
$$\begin{bmatrix} v_x \\ v_y \end{bmatrix}_O = \begin{bmatrix} 1 \\ 0 \end{bmatrix}_O \dot{q}_{1a} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}_O \dot{q}_{1p} = \overline{\mathbf{v}}_1 \quad \dot{q}_{1a} + \mathbf{v}_1 \quad \dot{q}_{1p}.$$

finger #2:
$$\begin{bmatrix} v_x \\ v_y \end{bmatrix}_O = \begin{bmatrix} -1 \\ 0 \end{bmatrix}_O \dot{q}_{2a} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}_O \dot{q}_{2p} = \overline{\mathbf{v}}_2 \quad \dot{q}_{2a} + \mathbf{v}_2 \quad \dot{q}_{2p},$$

considering just fingers 1 and 2...

$$V = \bigcap_{i=1}^{2} \mathbf{v}_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \bigcap \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

$\Longrightarrow$ fingers 1 and 2 alone do not fully immobilize the object

Grasp Analysis - Mobility and Connectivity

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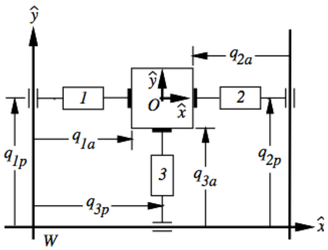


considering fingers 1, 2, and 3, the intersection of unrestricted object velocities is empty...

$$V = \bigcap_{i=1}^{3} \mathbf{v}_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \bigcap \begin{bmatrix} 0 \\ 1 \end{bmatrix} \bigcap \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \emptyset,$$

...these three (fixed) contacts fully immobilize the object,

and the union of velocity constaints derived from active degrees of freedom spans $\mathbb{R}^2$:

$$\overline{V} = \bigcup_{i=1}^{3} \overline{\mathbf{v}}_i = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \bigcup \begin{bmatrix} -1 \\ 0 \end{bmatrix} \bigcup \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \mathbb{R}^2$$

$\Longrightarrow$ the object position fully controllable in the (x, y) plane by the planar hand.

Grasp Analysis - Form Closure

Definition (Form Closure) - a condition of complete restraint in which any object twist $\in \mathbb{R}^6$ is inconsistent with rigid body assumption for objects and fixed contacts.

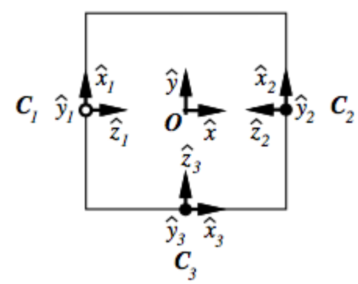
form closure can be defined solely in terms of mobility without specifying contact forces at all*

* form closure does not require friction

Grasp Analysis - Form Closure

- Reuleaux
  - planar bodies require at least four frictionless contacts for form closure in $\mathbb{R}^3$, and
  - *exceptional* surfaces exist for which form closure is impossible given any number of frictionless point contacts.
- Somoff (1897) proved that at least 7 frictionless point contacts are necessary for form closure in $\mathbb{R}^6$
- Mishra, Schwartz and Sharir (1987) - established an upper bound of 6 frictionless point contacts on planar objects with piecewise smooth contours, and 12 for the spatial case (except for Reuleaux's exceptional surfaces).

The Grasp Jacobian



$$\begin{bmatrix} v_{1x} \\ v_{1z} \\ v_{2x} \\ v_{2z} \\ v_{3x} \\ v_{3z} \end{bmatrix}_C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \end{bmatrix}_O$$

$$\boxed{\mathbf{v}_C = \mathbf{G}^T \mathbf{v}_O,} \; \textit{where } \mathbf{G} = [\mathbf{v}_1 \; \overline{\mathbf{v}}_1 \; \mathbf{v}_2 \; \overline{\mathbf{v}}_2 \; \mathbf{v}_3 \; \overline{\mathbf{v}}_3].$$

Wrenches and Twists

the power transmitted to the object by a contact is equal to the power generated by the contact forces:

$$\mathbf{w}_O^T \mathbf{v}_O = \mathbf{w}_C^T \mathbf{v}_C$$

but since , $\mathbf{v}_C = \mathbf{G}^T \mathbf{v}_O,$

$$\mathbf{w}_O^T \mathbf{v}_O = \mathbf{w}_C^T [\mathbf{G}^T \mathbf{v}_O],$$

$$\mathbf{w}_O^T = \mathbf{w}_C^T \mathbf{G}^T, \; or$$

$$\boxed{\mathbf{w}_O = \mathbf{G}\mathbf{w}_C.}$$

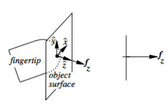
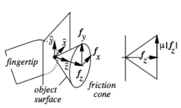
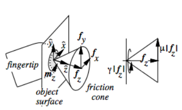
Wrenches and Twists

so, from our planar 3-contact example:

$$\mathbf{v}_C = \mathbf{G}^T \mathbf{v}_O, \qquad \begin{bmatrix} v_{1x} \\ v_{1z} \\ v_{2x} \\ v_{2z} \\ v_{3x} \\ v_{3z} \end{bmatrix}_C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \end{bmatrix}_O$$

$$\mathbf{w}_O = \mathbf{G}\mathbf{w}_C. \qquad \begin{bmatrix} f_x \\ f_y \end{bmatrix}_O = \begin{bmatrix} 0 & 1 & 0 & -1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_{1x} \\ f_{1z} \\ f_{2x} \\ f_{2z} \\ f_{3x} \\ f_{3z} \end{bmatrix}_C$$

Contact Types

contact type	geometry	selection matrix $\mathbf{H}^T$ $\mathbf{w}_C = \mathbf{H}^T \boldsymbol{\lambda}$	constraints
frictionless point contact		$\mathbf{w}_C = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} [\lambda_{fz}]$	$\lambda_{fz} \geq 0$
point contact with friction		$\mathbf{w}_C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_{fx} \\ \lambda_{fy} \\ \lambda_{fz} \end{bmatrix}$	$\lambda_{fz} \geq 0$ $[\lambda_{fx}^2 + \lambda_{fy}^2]^{1/2} \leq \mu\lambda_{fz}$
soft finger		$\mathbf{w}_C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda_{fx} \\ \lambda_{fy} \\ \lambda_{fz} \\ \lambda_{mz} \end{bmatrix}$	$\lambda_{fz} \geq 0$ $[\lambda_{fx}^2 + \lambda_{fy}^2]^{1/2} \leq \mu\lambda_{fz}$ $\lambda_{mz} \leq \gamma\lambda_{fz}$

Rotating Contact Wrenches

given the rotation matrix ${}_O\mathbf{R}_{Ci}$ that transforms vectors in contact frame i into object frame—the block diagonal

$$\overline{\mathbf{R}}_i = \left[\begin{array}{c|c} {}_O\mathbf{R}_{Ci} & \mathbf{0} \\ \hline \mathbf{0} & {}_O\mathbf{R}_{Ci} \end{array} \right]$$

applies this rotation to the force and moment components of the contact wrench independently.

Translating Contact Wrenches

- the force component of the wrench maps to the same forces in the object frame

Translating Contact Wrenches

- contact frame moments sum with the "couple" $\boldsymbol{\rho} \times \boldsymbol{f}_C$, where $\boldsymbol{\rho} \in \mathbb{R}^3$ is the position vector locating frame $\boldsymbol{C}$ with respect to frame $\boldsymbol{O}$

Translating Contact Wrenches

$$\mathbf{P}_i = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & -\rho_z & \rho_y & 1 & 0 & 0 \\ \rho_z & 0 & -\rho_x & 0 & 1 & 0 \\ -\rho_y & \rho_x & 0 & 0 & 0 & 1 \end{array}\right]$$

the product of matrix $\mathbf{P}_i$ with a wrench at the contact site transforms that wrench into the equivalent wrench at the object frame.

Constructing a Grasp Jacobian - Algebraically

$$(\mathbf{w}_O)_i = \mathbf{G}_i \mathbf{w}_{Ci} = \mathbf{G}_i \mathbf{H}_i^T \boldsymbol{\lambda}_{Ci}$$ contact by contact

$$(\mathbf{w}_O)_i = \mathbf{G}_i^* \boldsymbol{\lambda}_{Ci}, \quad \text{where,} \quad \mathbf{G}_i^* = \mathbf{P}_i \overline{\mathbf{R}}_i \mathbf{H}_i^T.$$

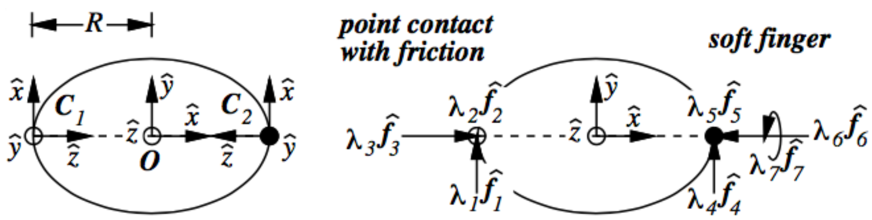
For an n contact grasp configuration, the grasp Jacobian and effort is written

$$\mathbf{G}^* = [\mathbf{G}_1^* \cdots \mathbf{G}_n^*]$$

and,

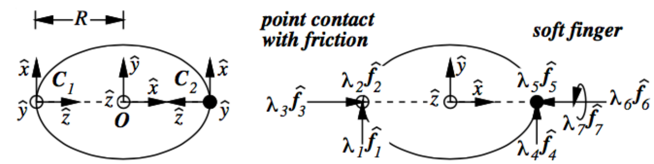
$$\boldsymbol{\lambda} = [\boldsymbol{\lambda}_{C1}^T \cdots \boldsymbol{\lambda}_{Cn}^T]^T.$$

Solving for Grasp Forces



assume that unit contact forces, $\boldsymbol{f}_i \in \mathbb{R}^3$, are independent

Solving for Grasp Forces - **by inspection**



$$\mathbf{w}_O = \overset{\begin{matrix}\mathbf{w}_1 & \mathbf{w}_2 & \mathbf{w}_3 & \mathbf{w}_4 & \mathbf{w}_5 & \mathbf{w}_6 & \mathbf{w}_7\end{matrix}}{\left[\begin{array}{ccc|cccc} 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & R & 0 & 0 & R & 0 & 0 \\ -R & 0 & 0 & R & 0 & 0 & 0 \end{array}\right]} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \\ \vdots \\ \\ \lambda_7 \end{bmatrix}$$

$$\mathbf{w}_O = \mathbf{G}^* \boldsymbol{\lambda}$$

Prehensile Grasp Stability - Force Closure

the ability of a contact configuration to suppress random disturbances by modifying grip forces

Definition (Force Closure) - A grasp is force closure if a solution for contact frame wrenches $\boldsymbol{\lambda}$ exists that complies with contact type constraints such that

$$\mathbf{G}^*\boldsymbol{\lambda} = \mathbf{w}_{dist} \quad \text{for arbitrary } \mathbf{w}_{dist}$$

$\Longrightarrow$ the contact configuration is capable of generating a convex envelope of grasp wrench responses (that contains the origin).

prehensile

Grasp Stability

...stated in another way...

$$\mathbf{w}_O = \mathbf{G}^*\boldsymbol{\lambda}$$

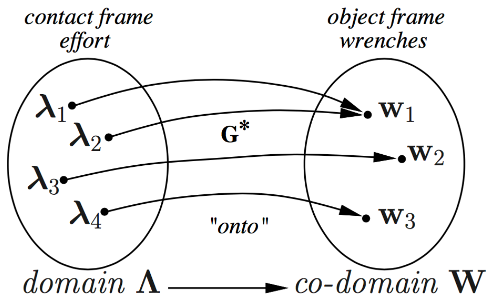
a grasp is force closure (and stabilizable) if and only if $\mathbf{G}^*$ is surjective [Murray, Li, Sastry 1994]

surjection (“onto”) - <u>every</u> object frame wrench $\mathbf{w}_i$ is accessible by applying transform $\mathbf{G}^*$ to <u>at least one</u> combination of contact frame effort $\boldsymbol{\lambda}$

Grasp Stability

surjection ("onto") - every object frame wrench $\mathbf{w}_i$ is accessible by applying transform $\mathbf{G}^*$ to at least one combination of contact frame effort $\boldsymbol{\lambda}$

many-to-one



$$\forall \mathbf{w}_i \in \mathbf{W} \;\; \exists \boldsymbol{\lambda} \in \boldsymbol{\Lambda} \;\; \textit{such that} \;\; \mathbf{w}_i = \mathbf{G}^*\boldsymbol{\lambda}$$

Solving for Grasp Forces

$$\mathbf{w}_O = \mathbf{G}^*\boldsymbol{\lambda} = \mathbf{G}^*(\boldsymbol{\lambda}_p + \boldsymbol{\kappa}^T\boldsymbol{\lambda}_h)$$

where solutions $\boldsymbol{\lambda}$ have homogeneous and particular parts,

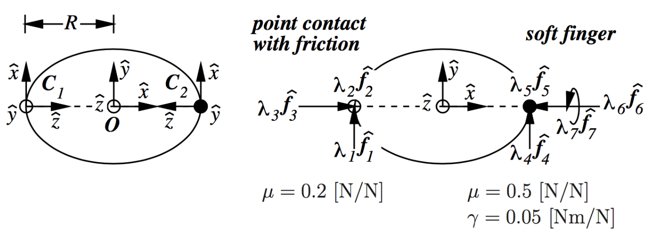
$$\boldsymbol{\lambda} = \boldsymbol{\lambda}_p + \boldsymbol{\kappa}^T\boldsymbol{\lambda}_h$$

$\boldsymbol{\lambda}_h$ is the **homogeneous part** of the solution and describes combinations of contact forces that impart zero net force to the object.

$$\mathbf{G}^*\boldsymbol{\lambda}_h = \mathbf{0}$$

- $\mathbf{G}^*$ must be full rank to achieve arbitrary reference wrenches
- $\boldsymbol{\lambda}$ must satisfy inequality constraints for unisense normal forces and contact friction.

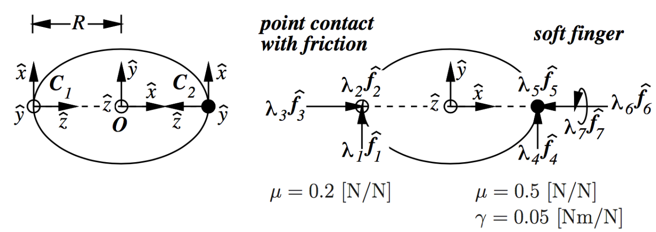
Solving for Grasp Forces



suppose: grasp forces must support an object load of $-1.0\hat{\boldsymbol{y}}$ $[N]$

$$M_x = 0 \Rightarrow \lambda_7 = 0$$

Solving for Grasp Forces



suppose: grasp forces must support an object load of $-1.0\hat{\boldsymbol{y}}$ $[N]$

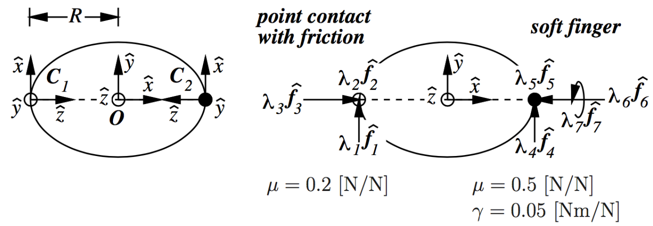
$$F_y = 1 \Rightarrow \quad \lambda_1 + \lambda_4 = 1$$
$$M_z = 0 \Rightarrow -\lambda_1 + \lambda_4 = 0$$

$$\Rightarrow \quad \lambda_1 = \lambda_4 = 0.5$$

particular solution

$\lambda_7 = 0$

Solving for Grasp Forces



suppose: grasp forces must support an object load of $-1.0\hat{\boldsymbol{y}}$ $[N]$

$\lambda_1 = 0.5$

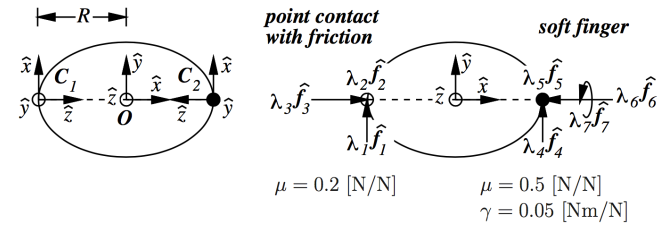
$F_x = 0 \Rightarrow \lambda_3 - \lambda_6 = 0$

$\lambda_4 = 0.5$

$\Rightarrow \quad \lambda_3 = \lambda_6$

$\lambda_7 = 0$

Solving for Grasp Forces



suppose: grasp forces must support an object load of $-1.0\hat{\boldsymbol{y}}$ $[N]$

$\lambda_1 = 0.5$

$\lambda_3 = \lambda_6$

$\lambda_4 = 0.5$

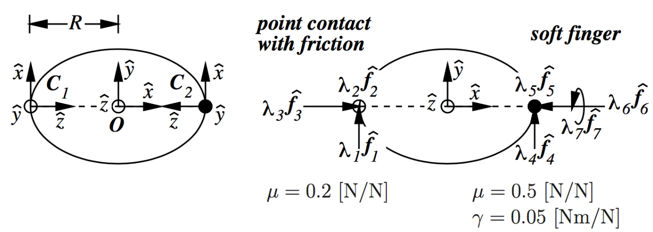
$\lambda_7 = 0$

$F_z = 0 \Rightarrow \lambda_2 - \lambda_5 = 0$

$M_y = 0 \Rightarrow \lambda_2 + \lambda_5 = 0$

$\Rightarrow \quad \lambda_2 = \lambda_5 = 0$

Solving for Grasp Forces



suppose: grasp forces must support an object load of $-1.0\hat{\boldsymbol{y}}$ $[N]$

$\lambda_1 = 0.5$

$\lambda_3 = \lambda_6$

$\lambda_4 = 0.5$

$\lambda_2 = \lambda_5 = 0$

$\lambda_7 = 0$

frictional constraints

$\lambda_1 \leq \mu\lambda_3$

$0.5 \leq (0.2)\lambda_3$

$\lambda_3 \geq 2.5$

Solving for Grasp Forces

$\lambda_1 = 0.5$

$\lambda_3 = \lambda_6$

$\lambda_4 = 0.5$

$\lambda_2 = \lambda_5 = 0$

$\lambda_7 = 0$

frictional constraints

$\lambda_1 \leq \mu\lambda_3$

$0.5 \leq (0.2)\lambda_3$

$\lambda_3 \geq 2.5$

$$\boldsymbol{\lambda} = \boldsymbol{\lambda}_p + \boldsymbol{\kappa}^T\boldsymbol{\lambda}_h = [0.5 \;\; 0 \;\; 0 \;\; 0.5 \;\; 0 \;\; 0 \;\; 0]_p^T + \kappa\,[0 \;\; 0 \;\; 1 \;\; 0 \;\; 0 \;\; 1 \;\; 0]_h^T$$

and, $\kappa \geq 2.5$ satisfies frictional constraints

automated techniques based on mathematical programming are used to solve these systems subject to contact type constraints

Force Closure Revisited

1. The grasped object is in quasistatic equilibrium, there are no net forces or moments,
2. all forces are applied within the cone of friction so that there is no slippage, and,
3. an externally applied force can be resisted by finger forces with a finite and controllable deflection.

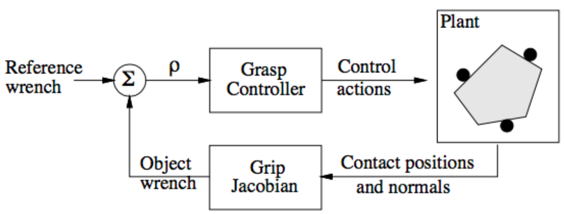
Grasp Synthesis

grasp analysis *vs*. grasp synthesis

once we have a grasp geometry,
grasp analysis provides a grasp force solution

but, how is a grasp geometry determined?

Closed-Loop, Sensor-Driven Grasp Control



$$\phi = \rho^T \mathbf{M} \rho = \left(\frac{1}{k}\sum_{i=1}^{k}\omega_i\right)^T \mathbf{M} \left(\frac{1}{k}\sum_{i=1}^{k}\omega_i\right)$$

the control Jacobian, $\mathbf{J}_c$, is the partial of ϕ with respect to contact coordinates $\boldsymbol{q}$

Navigation Function - Grasp *Control*

1. The grasped object is in quasistatic equilibrium, there are no net forces or moments,