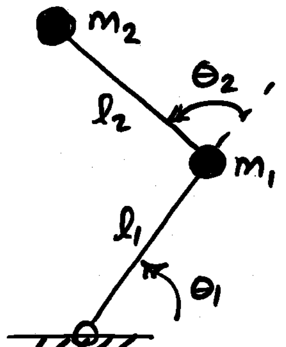
LAGRANGIAN - ROGER-THE-CRAB



DEFINITIONS:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}_{m_1} = J_1 \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \qquad J_1 = \begin{bmatrix} -l_1 s_1 & 0 \\ l_1 c_1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}_{m_2} = J_2 \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \qquad J_2 = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \end{bmatrix}$$

ABOUT THE CENTER OF MASS FOR EACH LINK:

$$\tilde{I}_1 = \tilde{I}_2 = 0 \qquad \text{POINT MASS}$$

KINETIC ENERGY:

$$T = \frac{1}{2}\left[\underbrace{(J_1\dot{\theta})^T m_1 (J_1\dot{\theta})}_{\dot{\theta}^T (J_1^T m_1 J_1)\dot{\theta}} + \cancelto{0}{\dot{\theta}^T \tilde{I}_1 \dot{\theta}} + \underbrace{(J_2\dot{\theta})^T m_2 (J_2\dot{\theta})}_{\dot{\theta}^T (J_2^T m_2 J_2)\dot{\theta}} + \cancelto{0}{\dot{\theta}^T \tilde{I}_2 \dot{\theta}} \right]$$

$$T = \frac{1}{2}\dot{\theta}^T \left[J_1^T m_1 J_1 + J_2^T m_2 J_2 \right] \dot{\theta}$$

KINETIC ENERGY:

$$T = \frac{1}{2}\begin{bmatrix}\dot{\theta}_1 & \dot{\theta}_2\end{bmatrix}\left[\underbrace{J_1^T m_1 J_1} + \underbrace{J_2^T m_2 J_2}\right]\begin{bmatrix}\dot{\theta}_1 \\ \dot{\theta}_2\end{bmatrix}$$

$$\begin{bmatrix} m_1 l_1^2 & 0 \\ 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} (m_2 l_1^2 + 2m_2 l_1 l_2 c_2 + m_2 l_2^2) & (m_2 l_1 l_2 c_2 + m_2 l_2^2) \\ (m_2 l_1 l_2 c_2 + m_2 l_2^2) & (m_2 l_2^2) \end{bmatrix}$$

$$T = \frac{1}{2}\begin{bmatrix}\dot{\theta}_1 & \dot{\theta}_2\end{bmatrix}\begin{bmatrix} a & b \\ b & c \end{bmatrix}\begin{bmatrix}\dot{\theta}_1 \\ \dot{\theta}_2\end{bmatrix} = \frac{1}{2}\left(a\dot{\theta}_1^2 + 2b\dot{\theta}_1\dot{\theta}_2 + c\dot{\theta}_2^2\right)$$

$$a = m_1 l_1^2 + m_2 l_1^2 + 2m_2 l_1 l_2 c_2 + m_2 l_2^2$$
$$b = m_2 l_1 l_2 c_2 + m_2 l_2^2$$
$$c = m_2 l_2^2$$

KINETIC ENERGY

POTENTIAL ENERGY:

$$\boxed{V = m_1 g l_1 s_1 + m_2 g (l_1 s_1 + l_2 s_{12})}$$

POTENTIAL ENERGY

LAGRANGIAN:

$$\boxed{L = T - V = \frac{1}{2}\left(a\dot{\theta}_1^2 + 2b\dot{\theta}_1\dot{\theta}_2 + c\dot{\theta}_2^2\right) - m_1 g l_1 s_1 - m_2 g (l_1 s_1 + l_2 s_{12})}$$

EQUATIONS OF MOTION:

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot\theta}\right] - \frac{\partial L}{\partial \theta} = \Upsilon$$

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot\theta_1}\right] = \frac{d}{dt}\left[\frac{1}{2}(2a\dot\theta_1 + 2b\dot\theta_2)\right] = \frac{da}{dt}\dot\theta_1 + a\ddot\theta_1 + \frac{db}{dt}\dot\theta_2 + b\ddot\theta_2$$

$$\frac{da}{dt} = \frac{d}{dt}\left[2m_2 l_1 l_2 c_2\right] = -2m_2 l_1 l_2 s_2 \dot\theta_2$$

$$\frac{db}{dt} = \frac{d}{dt}\left[m_2 l_1 l_2 c_2\right] = -m_2 l_1 l_2 s_2 \dot\theta_2$$

$$\frac{\partial L}{\partial \theta_1} = -m_1 g l_1 c_1 - m_2 g(l_1 c_1 + l_2 c_{12})$$

$$\therefore \tau_1 = -2m_2 l_1 l_2 s_2 \dot\theta_1 \dot\theta_2 + (m_1 l_1^2 + m_2 l_1^2 + 2m_2 l_1 l_2 c_2 + m_2 l_2^2)\ddot\theta_1$$
$$- m_2 l_1 l_2 s_2 \dot\theta_2^2 + (m_2 l_1 l_2 c_2 + m_2 l_2^2)\ddot\theta_2$$
$$+ m_1 g l_1 c_1 + m_2 g(l_1 c_1 + l_2 c_{12})$$

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot\theta_2}\right] = \frac{d}{dt}\left[\frac{1}{2}(2b\dot\theta_1 + 2c\dot\theta_2)\right] = \frac{db}{dt}\dot\theta_1 + b\ddot\theta_1 + \frac{dc}{dt}\dot\theta_2 + c\ddot\theta_2$$

$$\frac{db}{dt} = -m_2 l_1 l_2 s_2 \dot\theta_2$$

$$\frac{dc}{dt} = 0$$

$$\frac{\partial L}{\partial \theta_2} = \frac{\partial}{\partial \theta_2}\left[\cancel{\tfrac{1}{2}}\left[(\cancel{2}m_2 l_1 l_2 c_2)\dot\theta_1^2 + \cancel{2}(m_2 l_1 l_2 c_2)\dot\theta_1\dot\theta_2\right]\right] - m_2 g l_2 c_{12}$$

$$\therefore \tau_2 = m_2 l_1 l_2 s_2 \dot\theta_1^2 + (m_2 l_1 l_2 c_2 + m_2 l_2^2)\ddot\theta_1 + (m_2 l_2^2)\ddot\theta_2$$
$$+ m_2 g l_2 c_{12}$$

RESULTS – LAGRANGIAN FORMULATION

$$\tau = M\ddot{\theta} + V + G$$

$$M = \begin{bmatrix} (m_1 l_1^2 + m_2 l_1^2 + 2 m_2 l_1 l_2 c_2 + m_2 l_2^2) & (m_2 l_1 l_2 c_2 + m_2 l_2^2) \\ (m_2 l_1 l_2 c_2 + m_2 l_2^2) & (m_2 l_2^2) \end{bmatrix}$$

$$V = \begin{bmatrix} -2 m_2 l_1 l_2 s_2 \dot{\theta}_1 \dot{\theta}_2 - m_2 l_1 l_2 s_2 \dot{\theta}_2^2 \\ m_2 l_1 l_2 s_2 \dot{\theta}_1^2 \end{bmatrix}$$

$$G = \begin{bmatrix} m_1 g l_1 c_1 + m_2 g (l_1 c_1 + l_2 c_{12}) \\ m_2 g l_2 c_{12} \end{bmatrix}$$

procedure "arm_dynamics()"