1. Bandwidth and Sampling Frequency (25 pts)

(a) at what frequency (rad/sec) do you expect to find a peak in the coefficients of the Fourier series describing the sinusoidal signal illustrated?

(b) at what frequency (rad/sec) should you sample this sinusoidal signal in order to faithfully reproduce the signal?

(c) what is the bandwidth (the difference between the lowest frequency and the highest frequency in the signal) of the square wave illustrated?

2. Edge Operators (10 pts)
Edge operators are all approximations of the gradient of the image function

\[ \nabla f = \frac{\partial f}{\partial x} \hat{x} + \frac{\partial f}{\partial y} \hat{y} \]

For example, the Prewit operators are:

\[ \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix} \]

(a) Show why these operators compute approximations of the image gradient.

(b) Describe how these operators may also be interpreted as templates that correlate to oriented high frequency events.

(c) Explain how these operators incorporate low-pass filtering and why.
3. Edge Localization

The edge, \( f(x) \), is sampled to produce the data in the top panel.

(a) The first operator is the Prewit gradient operator and the second is the one dimensional Laplacian operator. Perform the convolutions indicated, enter the values in the boxes provided and plot the results on the corresponding graphs on the left.

(b) Write pseudocode that uses the 1D arrays \( f' \) and \( f'' \) to implementing the edge sharpening algorithm and simulate the execution of the algorithm on the data above to label a single edge pixel in image \( h(x) \).

(c) What is the real valued \( x \) coordinate of the apparent center of the edge?