UMassAmherst

# The Control Basis: An Action Architecture for Computational Development



UMassAmherst

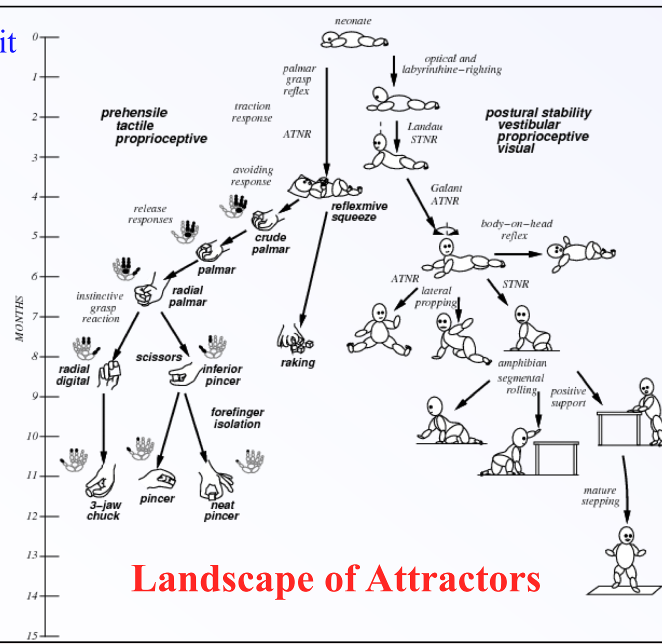
## Infant Sensorimotor Development - the First Year

incremental expression of knowledge in a common neurological framework

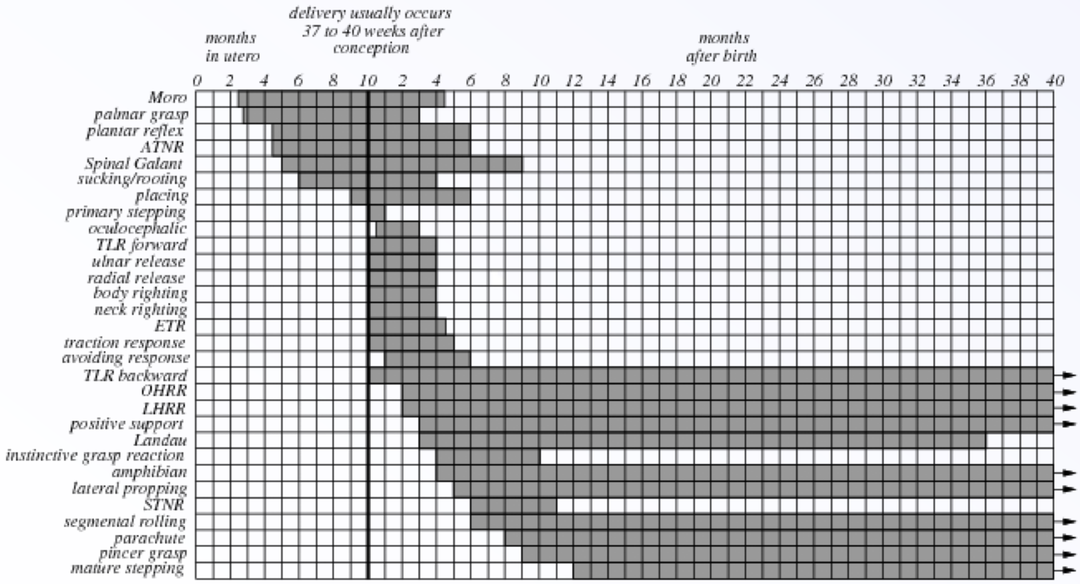
tacit

implicit

explicit



# Developmental Trajectory




…proximo-distal, cephalo-caudal, quasistatic-dynamic





# The Control Basis

In place of a relatively small set of special purpose developmental reflexes, an exhaustive array of closed-loop control relations is proposed that tile a high dimensional state space with multiple lower-dimensional attractors.

proximo/distal cephalon/caudal quasi-static/dynamic

the landscape of attractors is modeled as a discrete-event dynamical system within which the robot designer can overlay a time-varying system of logical constraints on the learner to support exploration-based developmental learning algorithms.

## Potential Functions

- The value of a scalar potential at the location of a particle in a field represents the energy that will be liberated if the particle is released from this configuration.

e.g. the gravitational potential of a particle of mass *m* near the Earth is the work required to move particle from the surface of the Earth to altitude *h*.

$$\phi_g = \int \mathbf{F} d\mathbf{s} = \int_0^h (mg) dz = mgh$$

The gradient of the potential field defines a force acting on the particle that returns the system to its equilibrium state.

$$\mathbf{F}_g = -\nabla mgh = -(mg)\,\hat{\mathbf{z}}$$





## Potential Functions – Spring-Mass-Damper

- For the SMD, the potential function is the energy stored in the spring

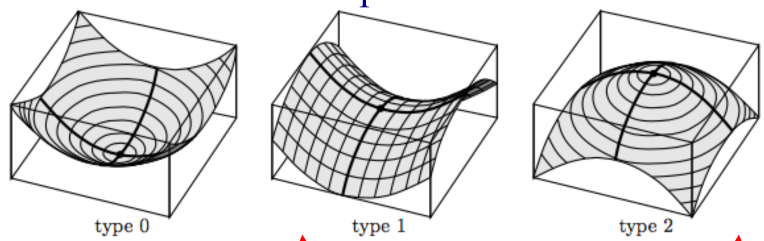
$$\phi_K = \int \mathbf{F} d\mathbf{s} = \int_0^x (Kx) dx = \frac{1}{2} K x^2$$

which is released when the spring is allowed to assume its original shape

$$\mathbf{F}_K = -\nabla \phi = -(Kx)\,\hat{\mathbf{x}}$$ Hooke's law

## Equilibrium Point Theory - Differential Geometry

minima  saddle point  maxima




stable *fixed* points  unstable critical points

Critical points – places where the gradient vanishes

$$\nabla \phi = \begin{bmatrix} \frac{\partial \phi}{\partial q_0} & \frac{\partial \phi}{\partial q_1} & \cdots & \frac{\partial \phi}{\partial q_n} \end{bmatrix} = \mathbf{0}.$$





## Potential Functions and Local Minima

Curvature in the Neighborhood of a Critical Point

$$\frac{\partial^2 \phi}{\partial \boldsymbol{q}^2} = \begin{bmatrix} \frac{\partial^2 \phi}{\partial q_1^2} & \frac{\partial^2 \phi}{\partial q_1 \partial q_2} & \cdots & \frac{\partial^2 \phi}{\partial q_1 \partial q_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 \phi}{\partial q_n \partial q_1} & \frac{\partial^2 \phi}{\partial q_n \partial q_2} & \cdots & \frac{\partial^2 \phi}{\partial q_n^2} \end{bmatrix}$$

a critical point is said to be **degenerate** if it also has zero curvature

excluding degenerate critical points, gradient descent will converge to type 0 critical points exclusively

## Potential Functions and Local Minima

$$\frac{\partial^2 \phi}{\partial \boldsymbol{q}^2} = \begin{bmatrix} \frac{\partial^2 \phi}{\partial q_1^2} & \frac{\partial^2 \phi}{\partial q_1 \partial q_2} & \cdots & \frac{\partial^2 \phi}{\partial q_1 \partial q_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 \phi}{\partial q_n \partial q_1} & \frac{\partial^2 \phi}{\partial q_n \partial q_2} & \cdots & \frac{\partial^2 \phi}{\partial q_n^2} \end{bmatrix}$$

**convex** – if the Hessian of $\phi$ is positive semi-definite over domain **q**, it has $\leq$ 1 stable fixed points on the interior of **q**

**harmonic** – if the trace of the Hessian then $\phi$ has no local minima

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial q_1^2} + \frac{\partial^2 \phi}{\partial q_2^2} + \cdots + \frac{\partial^2 \phi}{\partial q_n^2} = 0.$$





## Harmonic Functions

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial q_1^2} + \frac{\partial^2 \phi}{\partial q_2^2} + \cdots + \frac{\partial^2 \phi}{\partial q_n^2} = 0.$$

soap films, laminar fluid flow, steady state temperature in thermally conductive media, voltage distribution in electrically conductive media,

- exclude local minima (and maxima)
- only type 1 critical points (saddle points) (sets of measure zero)
- gradient flow produces non-intersecting *streamlines*
- *hitting probability* of a random walk --- use in path planning

# Navigation Functions

**analyticity** - infinitely differentiable ($C^\infty$ continuous) such that its *Taylor series* about $q_0$ converges to $\phi$ (q) for q in the neighborhood of $q_0$.

**polar** - gradients (streamlines) terminate at a unique minimum.
*functions that contain type 1 minima exclusively are polar*

**Morse** – functions whose isolevel curves are single points, closed curves, or closed curves that join at critical points …
*Morse functions cannot include degenerate critical points*

**admissibility** - Potential fields for robot control require *bounded torque* at obstacle boundaries (and everywhere else in the interior subset of configuration space as well).





# Control Basis: TRACK primitive

*objectives* x *sensors* x *effectors*

$\phi$ $\sigma$ $\tau$

navigation function

T: TRACK

$$a = \phi|_{\tau}^{\sigma}$$

**action:** closed-loop feature ($\sigma$) tracker where sensor viewpoint is controlled with kinematic chain ($\tau$)

visual foveation – contact force tracking
*any feature of any signal*

# Quasistatic-Dynamic Shaping

**state:** $\gamma(a) = \{ p(m_0)\ \ p(m_1)\ \ p(m_2)\ \ p(m_3) \}$

but the infant robot can bootstrap skills and accumulate models *at the same time* by using a quasi-static subset of models

$$\gamma(\phi, \dot{\phi}) = \begin{cases} \text{NO\_REFERENCE} : & \sigma \text{ undetected} \\ \text{TRANSIENT} : & \mid \mathbf{J}_c \mid > \epsilon \quad (\text{or } \nabla\phi > 0) \\ \text{CONVERGED} : & \mid \mathbf{J}_c \mid \leq \epsilon. \quad (\text{or } \nabla\phi = 0) \end{cases}$$





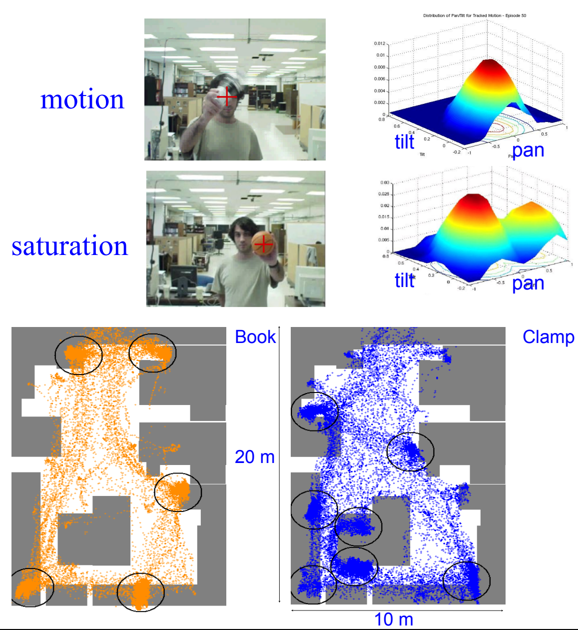
# Control Basis: SEARCH primitive

S: SEARCH

$$a = \phi|_{\tau}^{\tilde{\sigma}}$$

$$\tilde{\sigma} = Pr(\boldsymbol{u}_\tau | \gamma(\phi|_\tau^\sigma) \Rightarrow conv)$$

(TRACK)




the *orient* counterpart of TRACK actions

# EXAMPLE: Coordinated Human-Robot Search

on average, HR team performed 40% better than human alone





# SEARCHTRACK( ) revisited

$$\gamma(\phi, \dot{\phi}) = \begin{cases} \text{NO\_REFERENCE} : & \sigma \text{ undetected} \\ \text{TRANSIENT} : & |\mathbf{J}_c| > \epsilon \quad (\text{or } \nabla\phi > 0) \\ \text{CONVERGED} : & |\mathbf{J}_c| \leq \epsilon. \quad (\text{or } \nabla\phi = 0) \end{cases}$$

FOVEATE(eyes+base)

Actions = { SEARCH( ), TRACK( ) };

SEARCH(search-rec-delta-setpoints[8]) *internal state used to re-sample

$\gamma_T$ = TRACK(track-rec-delta setpoints[8]); state used for **control decisions**

```
switch( γT ) {
   case(0): setpoints[8]  += search-rec-delta-setpoints[8];
            break;
   case(1):
   case(2): setpoints[8]  += track-rec-delta-setpoints[8];
            break;
}
```

* contrary to the programming project directions **AFSM

UMassAmherst

# CHASETOUCH( )

how many TRACK-ers are there?

```
Actions = { SEARCHTRACK( ), MoveTo( ), PUNCH( )};
MOVETO(moveto-rec-delta-setpoints[8]);
PUNCH(punch-rec-delta-setpoints[8]);
γST = SEARCHTRACK(searchtrack-rec-delta-setpoints[8]);
switch( γT ) {
   case(0): setpoints[8]  += …;
            break;
   case(1):
   case(2): setpoints[8]  += …;
            break;
}
```

if actions produce disjoint rec-delta-setpoints[8], then they can be run concurrently



UMassAmherst

# Summary: Control Basis TRACK Actions and States

## Actions: the Control Jacobian

$$\mathbf{J}_c = \frac{d\phi(\sigma)}{d\boldsymbol{u}_\tau} = \left[ \frac{\partial\phi(\sigma)}{\partial u_1} \;\; \frac{\partial\phi(\sigma)}{\partial u_2} \;\; \cdots \;\; \frac{\partial\phi(\sigma)}{\partial u_n} \right]_{1\times n}$$

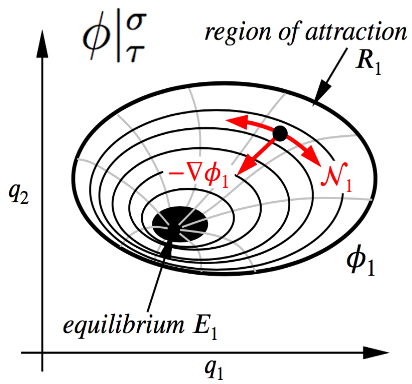
$$\begin{aligned} \Delta\boldsymbol{u}_\tau &= \kappa\, \mathbf{J}_c^{\#}\, (\phi_{ref} - \phi(\sigma)), \qquad \text{and if } \theta_{ref} = 0, \\ &= -\kappa\, \mathbf{J}_c^{\#}\, \phi(\sigma), \end{aligned}$$

## States: Quasi-Static Membership Function

$$\gamma(\phi, \dot{\phi}) = \begin{cases} \text{NO\_REFERENCE} : & \sigma \text{ undetected} \\ \text{TRANSIENT} : & |\,\mathbf{J}_c\,| > \epsilon \quad (\text{or } \nabla\phi > 0) \\ \text{CONVERGED} : & |\,\mathbf{J}_c\,| \leq \epsilon. \quad (\text{or } \nabla\phi = 0) \end{cases}$$

# Multi-Objective Search&TRACK Control

$$\begin{aligned}\Delta \mathbf{u}_\tau &= \mathbf{J}_1^{\#} \Delta\phi_1(\sigma_1) + \mathcal{N}_1 \left(\mathbf{J}_2^{\#} \Delta\phi_2(\sigma_2)\right) \\ &= \mathbf{J}_1^{\#} \Delta\phi_1(\sigma_1) + [\mathbf{I} - \mathbf{J}_1^{\#}\mathbf{J}_1] \left(\mathbf{J}_2^{\#} \Delta\phi_2(\sigma_2)\right)\end{aligned}$$



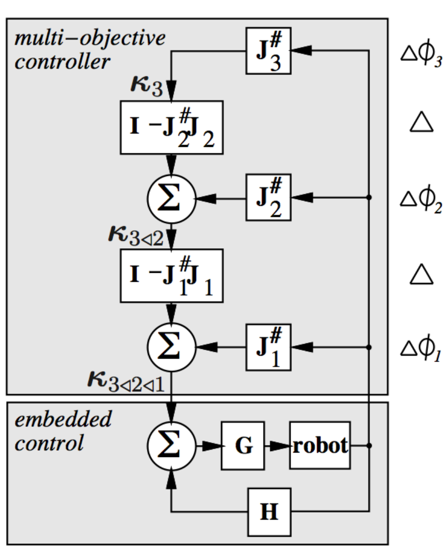

where, $\mathcal{N}_1 = [\mathbf{I} - \mathbf{J}_1^{\#}\mathbf{J}_1]$

the annihilator of J
Appendix A.9





# Multi-Objective Control

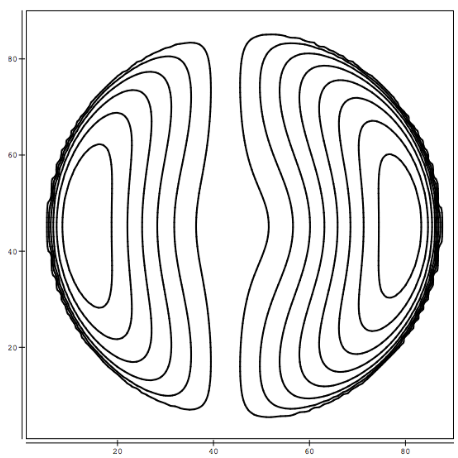


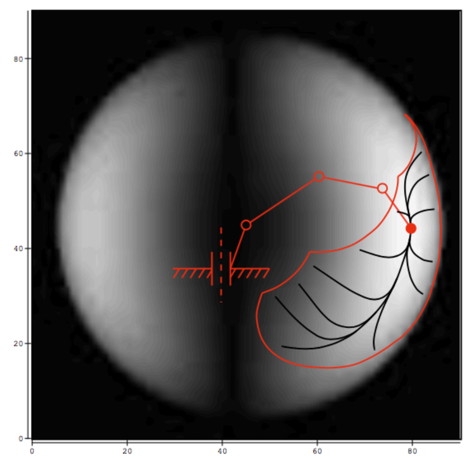

$$\boldsymbol{\kappa}_3 = \mathbf{J}_3^{\#}\Delta\phi_3$$

$$\boldsymbol{\kappa}_{3\triangleleft 2} = \mathbf{J}_2^{\#}\Delta\phi_2 + (\mathbf{I}_3 - \mathbf{J}_2^{\#}\mathbf{J}_2)\boldsymbol{\kappa}_3$$

$$\boldsymbol{\kappa}_{3\triangleleft 2\triangleleft 1} = \mathbf{J}_1^{\#}\Delta\phi_1 + (\mathbf{I}_3 - \mathbf{J}_1^{\#}\mathbf{J}_1)\boldsymbol{\kappa}_{3\triangleleft 2}$$

## Sidebar: there is also a POSTURAL primitive



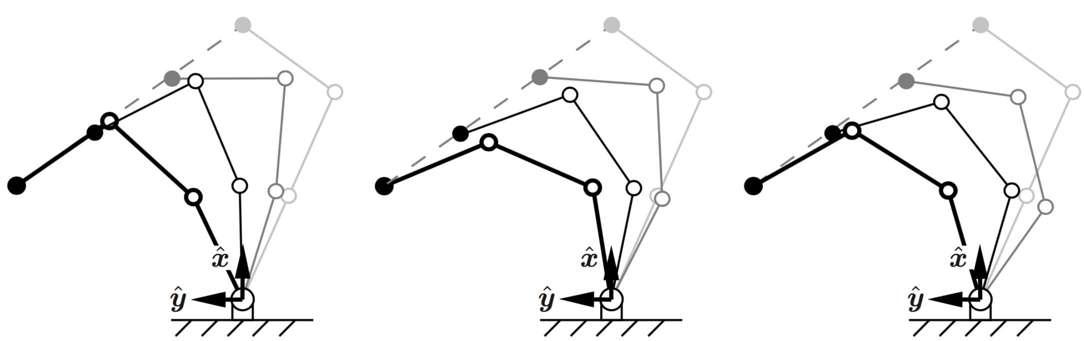


$$\phi_m = \phi(x, y) = \max_{\boldsymbol{\theta}} \sqrt{det(\mathbf{J}\mathbf{J}^T)}$$





## Sidebar: Combining POSTURAL and TRACK actions



(a) $\phi_{xy}$

Equation 10.10

(b) $\phi_{rom} \triangleleft \phi_{xy}$

(c) $\phi_m \triangleleft \phi_{xy}$

tendon routing in the human finger

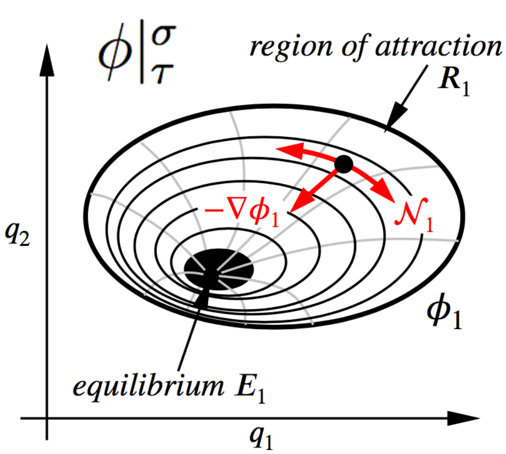
# ControlBasis-II

- more on SEARCH/TRACK/POSTURAL control
- modeling environmental control affordances
- grasp coffee cup example
- visual inspection task





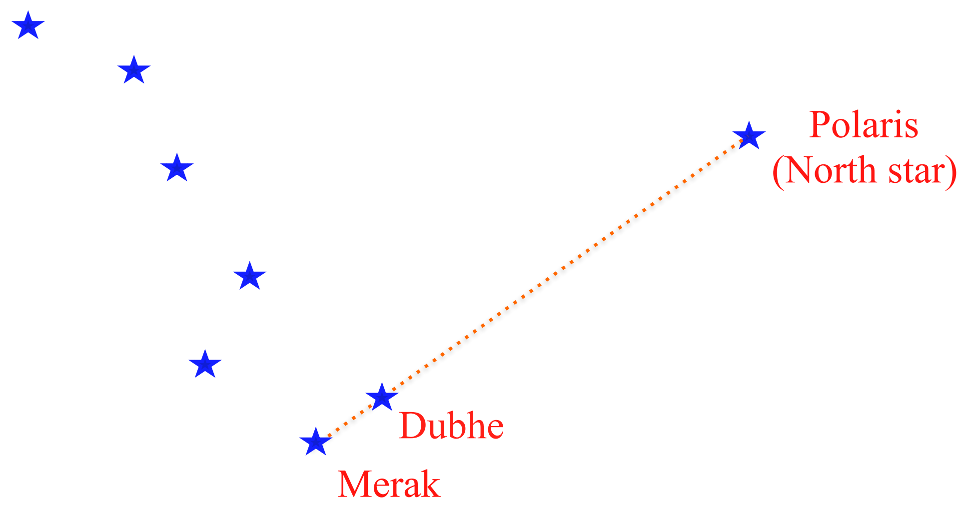
# Action Representation




Parametric Landscape of Attractors

- actions re-code the state space using classifiers in phase portrait
- "funneling" - fixed point sensory geometry relative to features *afforded* by the environment

# Landscapes of Attractors

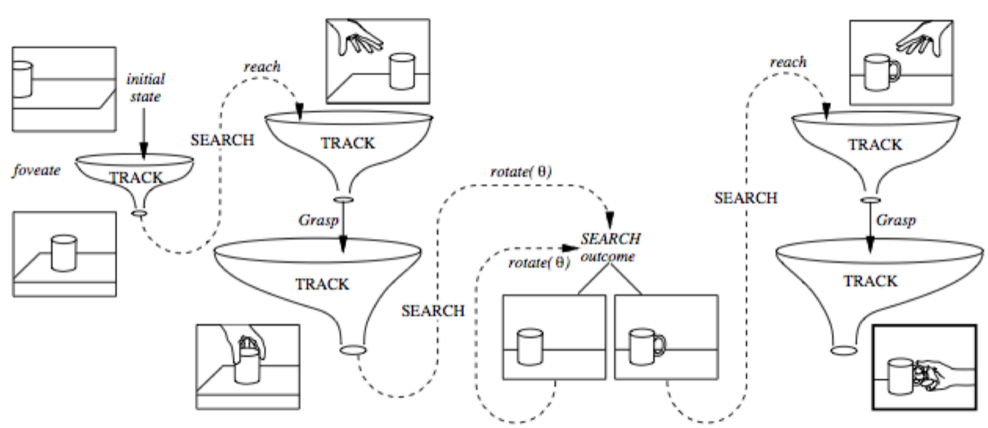







# *Funnel-ing* the World Using TRACK/SEARCH Actions

Grasp the cup's handle with the right hand.




**Models** - learn the parameters of SEARCH actions that transition between multiple TRACK affordances

UMassAmherst

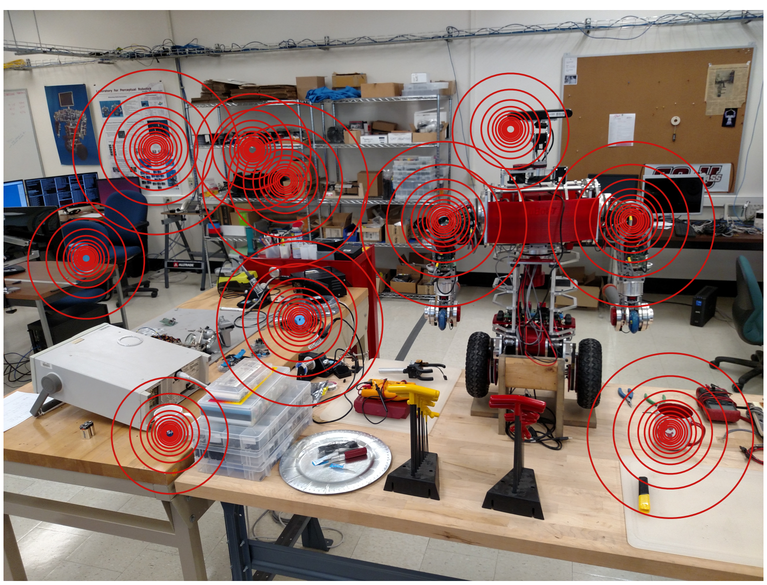
# Landscapes of Attractors

Where's my coffee cup?



UMassAmherst

# Landscapes of Attractors



this constellation of features…

UMassAmherst

# Landscapes of Attractors – visual features

serve as goals for oculomotor controls

parse the scene using geometric structure in constellations of visual features



UMassAmherst

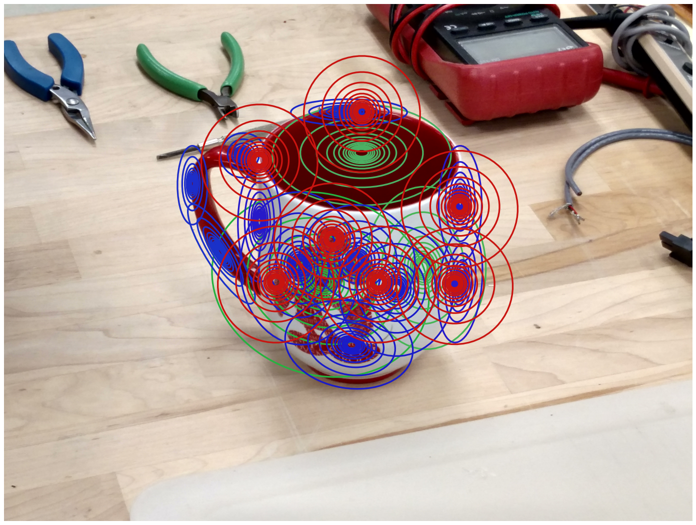
# Landscapes of Attractors - tactile

serve as goals for arm/hand controls

parse the scene using geometric structure in constellations of tactile features

# Landscapes of Attractors – multimodal

supports multiple recognition strategies that use different combinations of sensor modalities



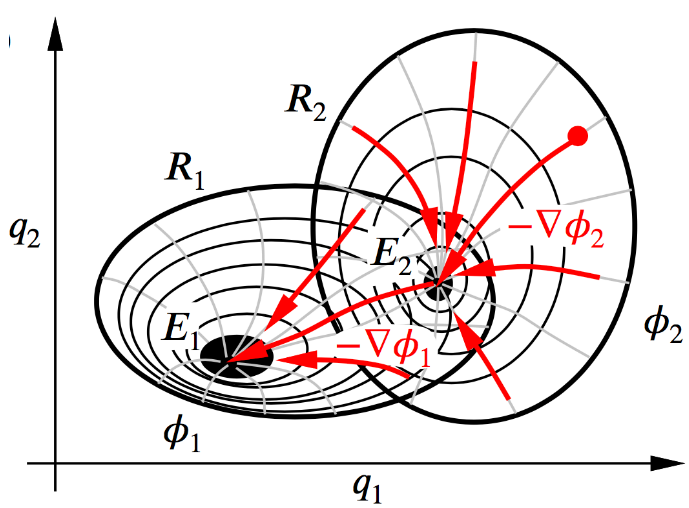
a multimodal goal set defining my coffee cup

parse the scene using geometric structure in constellations of multi-modal features

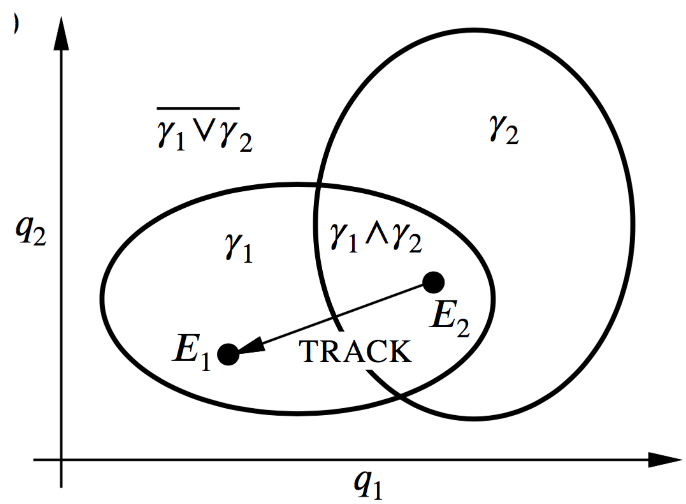




# Tiling the State Space with Skills



# Tiling the State Space with Skills

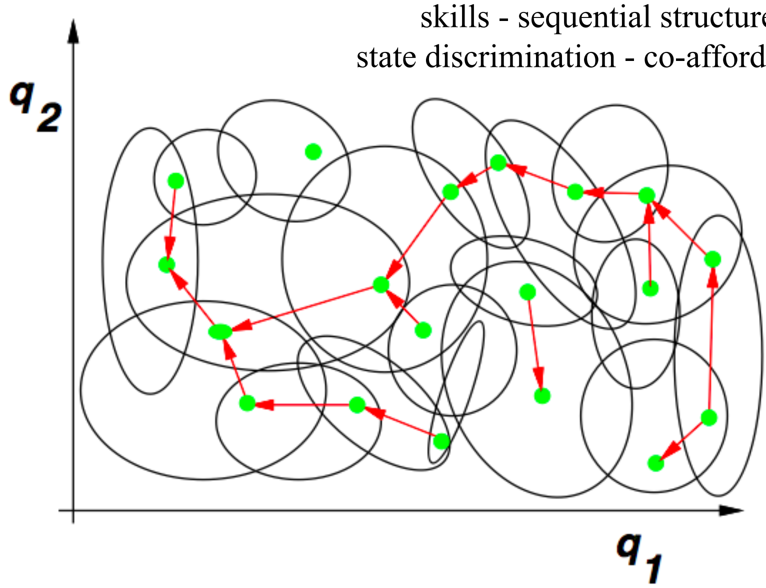





UMassAmherst

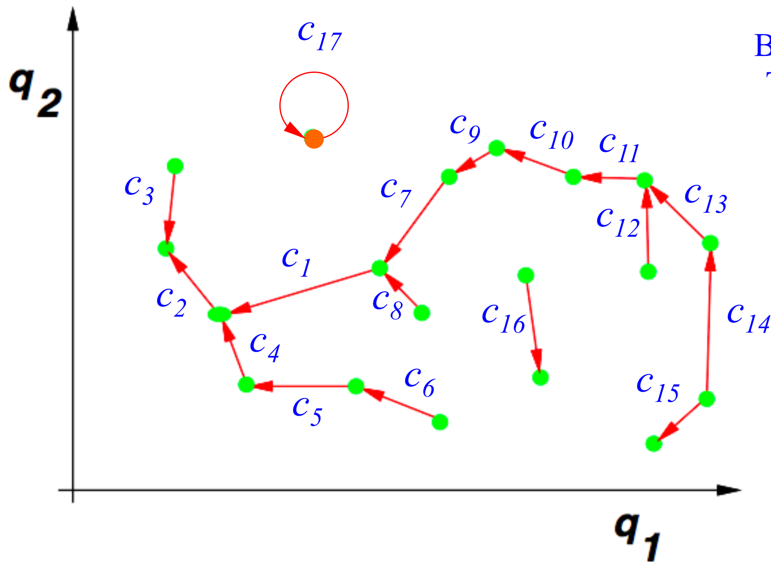
# Tiling the State Space with Skills

skills - sequential structure
state discrimination - co-affordances



# Tiling the State Space with Skills

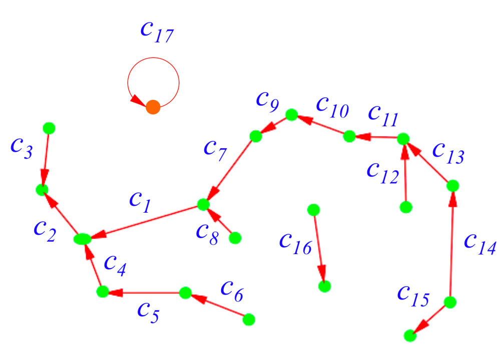
LMT 1984
Burridge 1999
Tedrake 2009








# Tiling the State Space with Skills



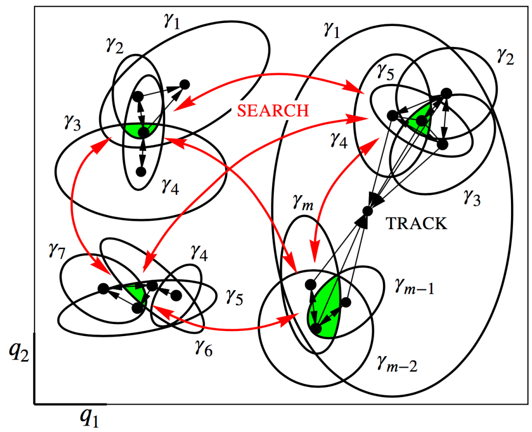

skills - sequential structure
state: [ $\gamma_1 \ldots \gamma_n$ ]

"objects": transition dynamics
reference to places $(q_1,q_2)$ not required

some of these are referenced to stimuli in the environment that move in (semi)rigid groups … "objects," "rooms," etc

joint distributions/graph homomorphisms
convey important context information

# Tiling the State Space with Skills




some of these are referenced to stimuli in the environment that move in (semi)rigid groups … "objects," "rooms," etc

joint distributions/graph homomorphisms
convey important context information





## sequential, multi-objective control

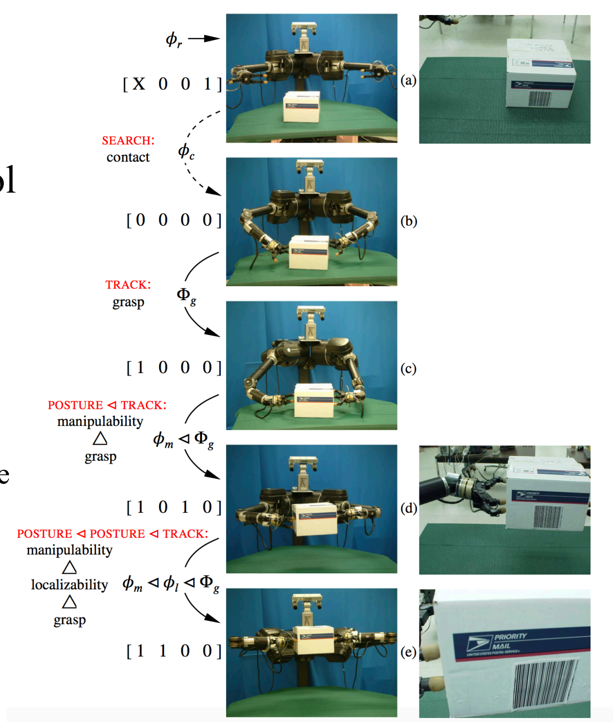
$\varphi_r$: the Postural range of motion objective;

$\varphi_c$ : Search for contact signals;

$\Phi_g$: a grasp control policy

$\varphi_m$: a Postural bimanual manipulability of the arms; and

$\varphi_l$: a Postural localizability controller



# Visual Conditioning