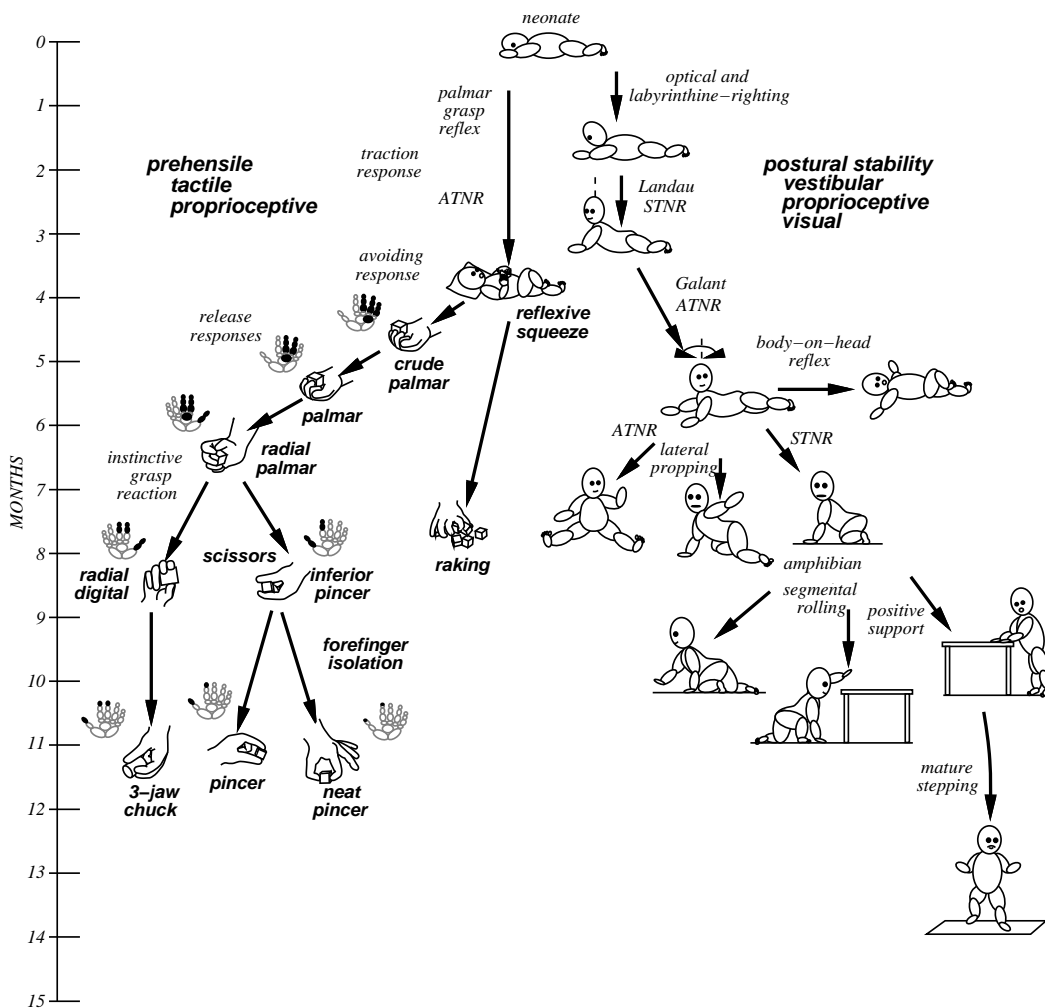
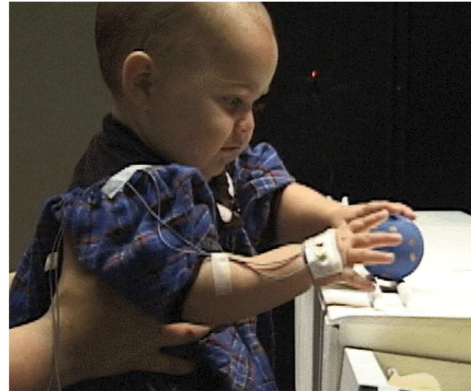


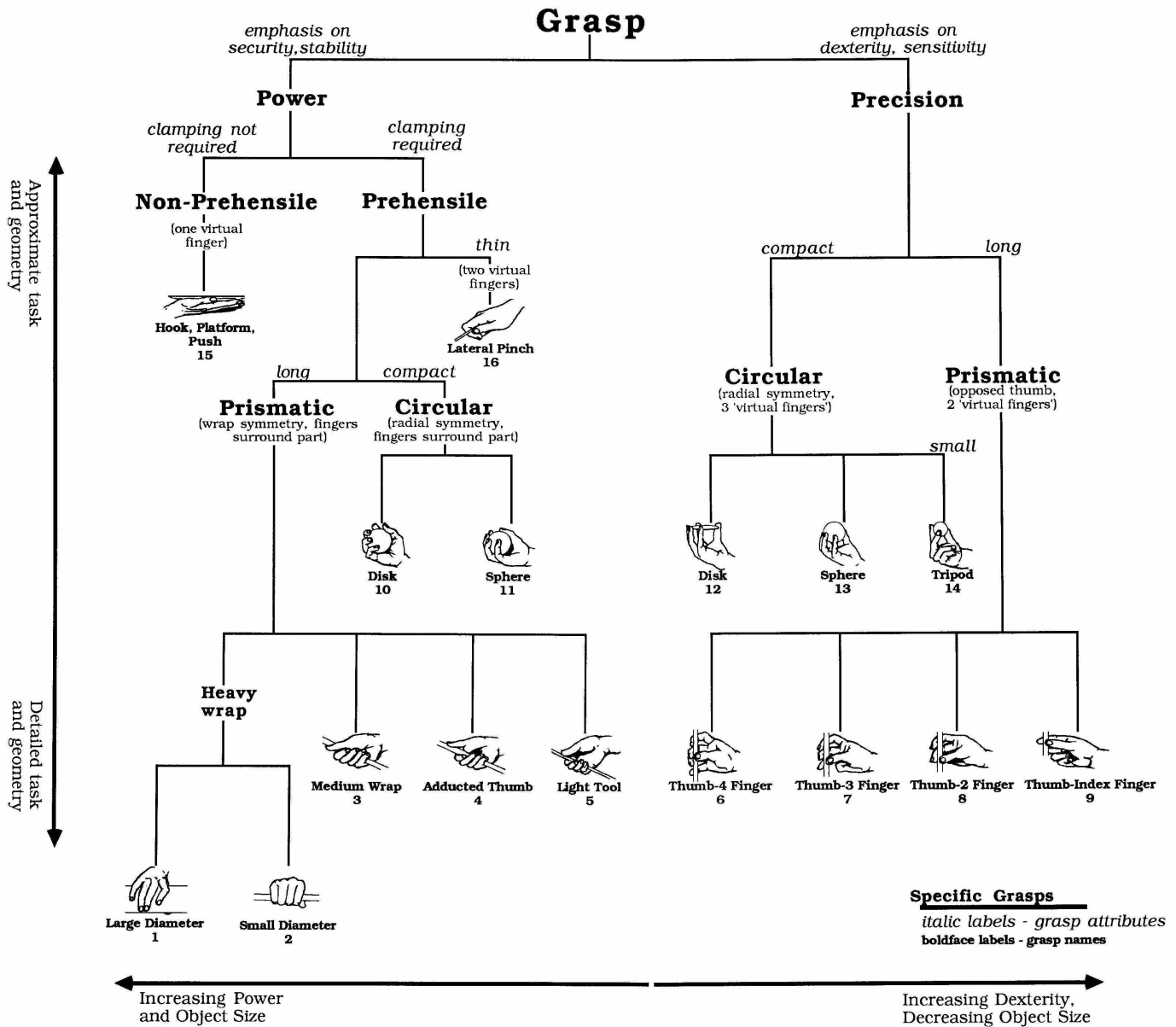


Grasping and Manipulation





Grasping and Manipulation





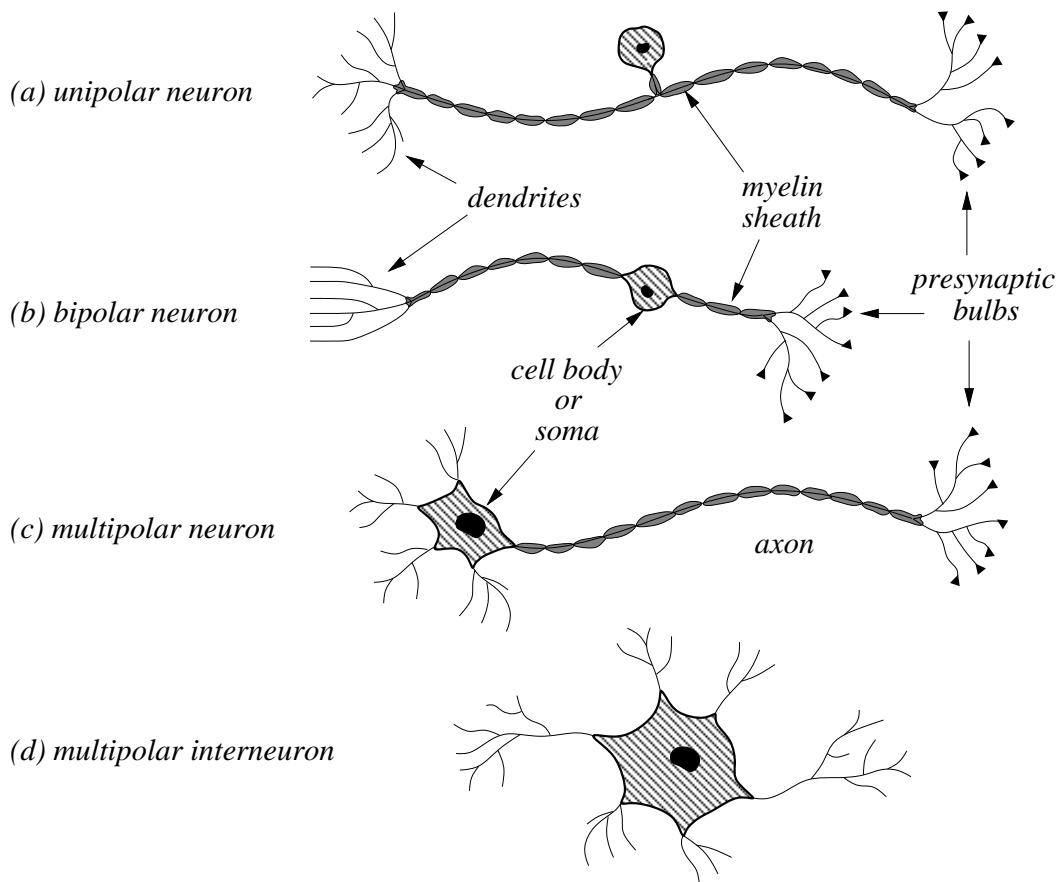
Haptics

- concerns the sense of touch—in particular, the perception and manipulation of objects using tactile and proprioceptive feedback gathered from peripheral mechanoreceptors, kinematic and dynamic state, muscle dynamics, neural conduction, and hierarchical processing
- incorporates a wide variety of central and peripheral mechanoreceptors and neural systems that measure forces, heat flux, pain, accelerations, the degree of stretch in muscle fibers and tendons.
- the result is high-fidelity perceptual information regarding force, contact and movement, sense of shape, hardness, texture, heat flux, grasp stability, and a variety of other subjective sensations associated with contact phenomena



Multi-Polar Neuron

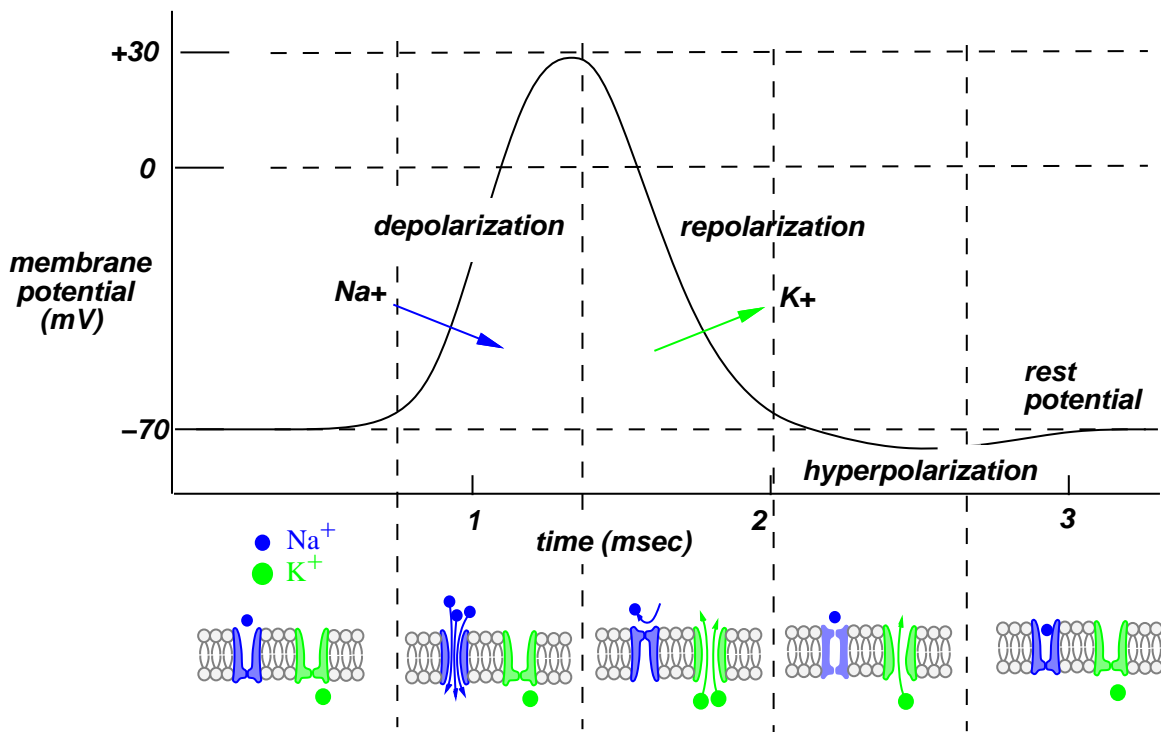
...the ancient Greeks observed that muscles could be permanently disabled by severing a thin white cord called a (peripheral) *nerve* that began and ended at the spinal cord





Intercellular Communication

indirect electrical coupling
refractory period
sodium and potassium pumps

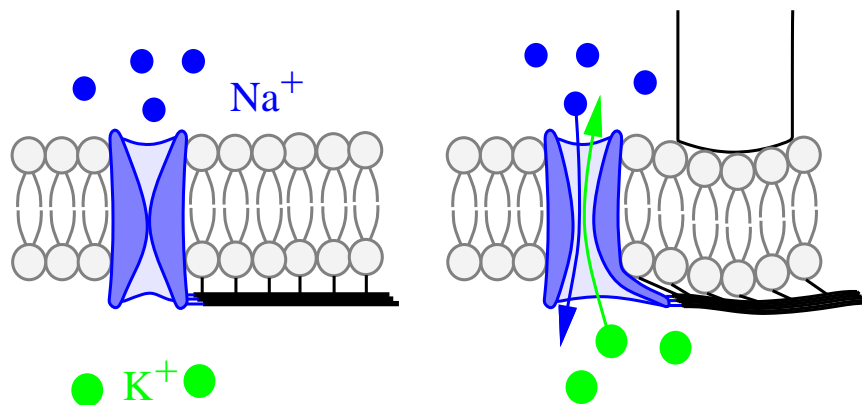


there are places (e.g. in substantia nigra) where ions are exchanged directly (cytoplasm to cytoplasm) between cell bodies.



Mechanoreceptors

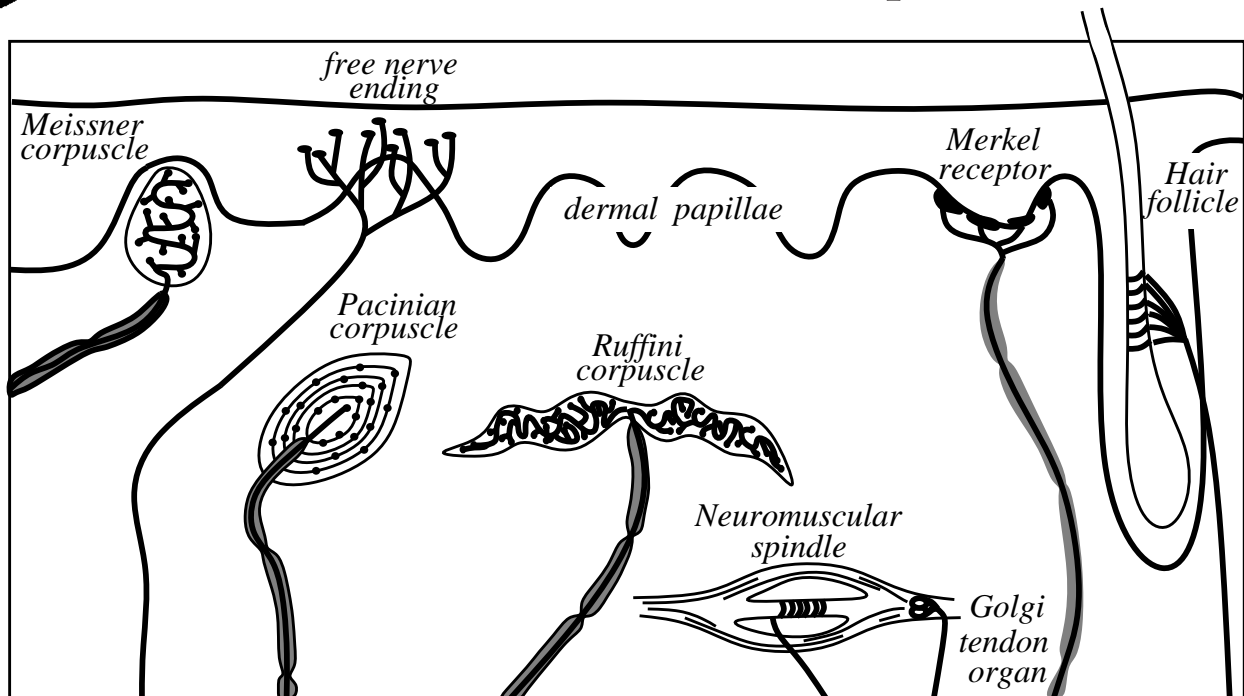
stressors come in many forms:
electrical, electromagnetic, and mechanical





The Peripheral Nervous System

Cutaneous Mechanoreceptors



- variable response and sensitivity, massive redundancy
- *touch blend* interpretation over multi-sensory signals temperature, pressure, and vibration ... distinguish wet, slimy, greasy, syrupy, mushy, doughy, gummy, spongy or dry, hardness, texture, compliance, size, shape, and curvature.
- movement is critical to the formation of haptic images leading to active tactile exploratory strategies



Biological Sensor Performance

frequency response	0 to 400 <i>Hz</i> (+ very high freq)
sensitivity	approx. 0.2 <i>grams/mm²</i>
max. response	100 <i>grams/mm²</i> ⇒ 55dB dynamic range*
spatial resolution	1.8 <i>mm</i> (two-point discrimination tests)
signal propagation	motor neurons 100 <i>m/sec</i> sensory neurons 2 to 80 <i>m/sec</i> autonomic neurons 0.5 to 15 <i>m/sec</i>

* - dynamic range = $20\log_{10}\left(\frac{P_{max}}{P_{min}}\right)$

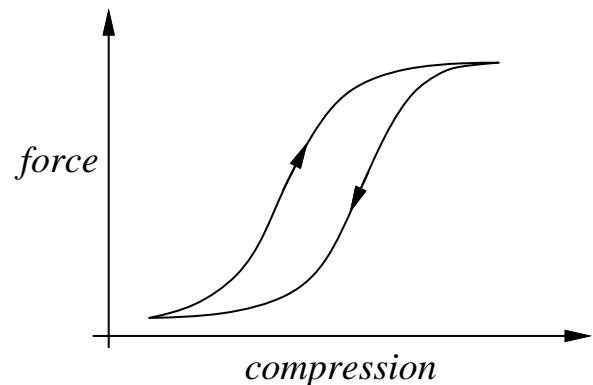
sound: ~ 100 dB \Rightarrow factor of 100,000 in amplitude and 10,000,000,000 in power

sight: ~ 90 dB \Rightarrow factor of 1,000,000,000 in brightness.



Robot Tactile Sensors - Desiderata

- contour vs. force sensing
- spatial resolution
- sensitivity - minimum magnitude of input signal required to produce a specified output signal-to-noise
- dynamic range - the ratio of largest to smallest detectable values

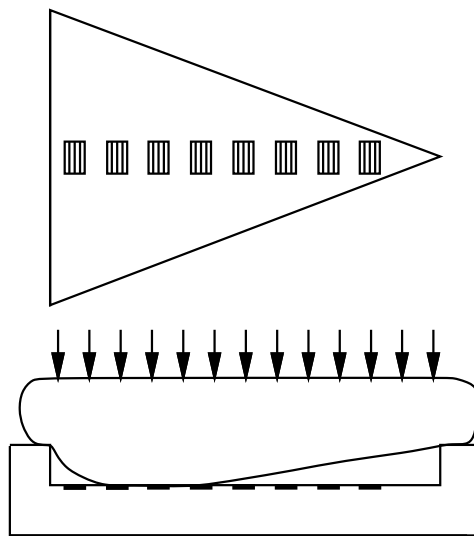


- hysteresis - “history” affect, plasticity
- frequency response (slip detection)
- manufacturing, durability, packaging, addressing, # wires



Binary Contact Switch

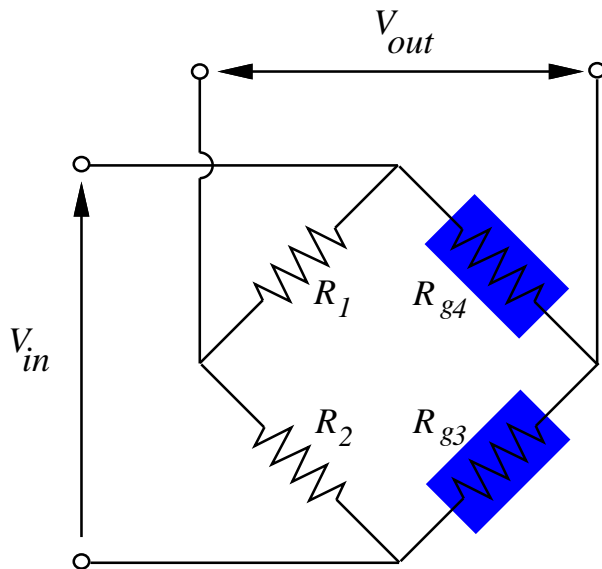
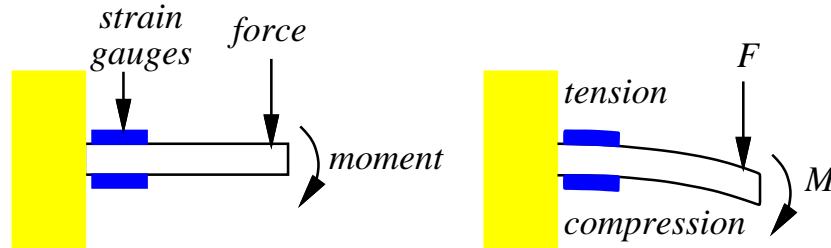
on/off contact switch, can be augmented easily to improve resolution...



Raibert (1984) increasing contact force threshold for each successive switch a prototype was constructed to produce 4 bits of pressure output per cell, serialized I/O, 200 tactile cells with a 1 *mm* spacing driven by 5 wires: power, ground, clock, data-in, and data-out.



Load Cells



"half bridge"

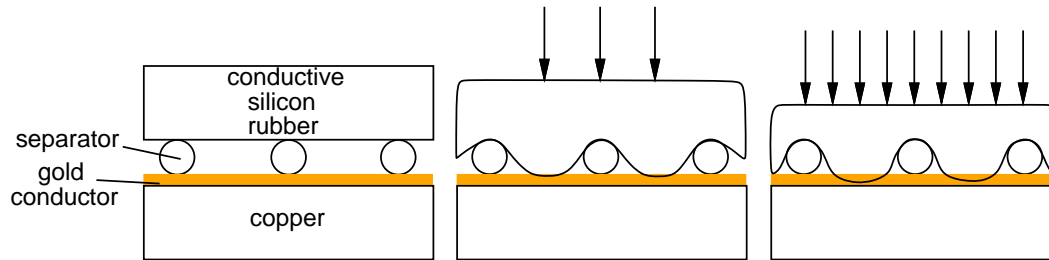
$$V_{out} = V_{in} \left[\frac{R_{g3}}{R_{g3} + R_{g4}} - \frac{R_2}{R_1 + R_2} \right]$$

Six-Axis Force/Torque Sensor: typically n strain gauges mounted on a multi-axial elastic element that measure multiple independent loads.

A (linear) calibration matrix maps signals to forces and moments $f_x, f_y, f_z, \tau_x, \tau_y, \tau_z$, and known sensor geometry can be used to compute contact positions and normals.



Conductive Elastomers



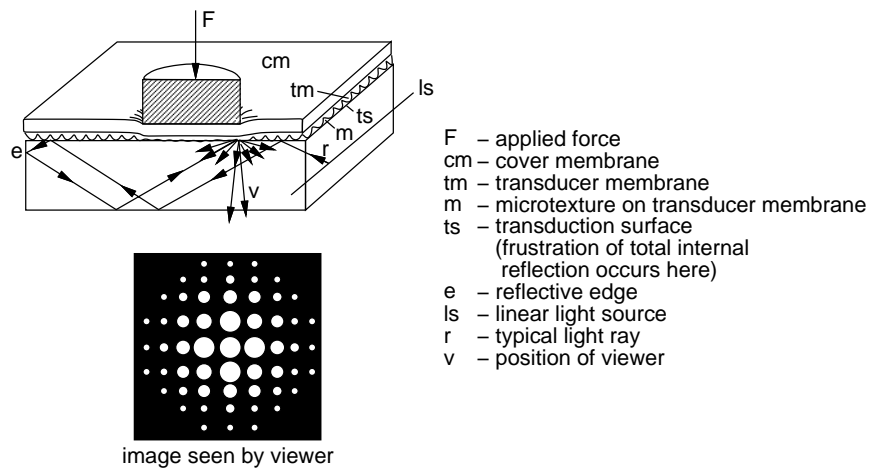
Hillis (1984)

- the separator is used to tune sensitivity, resolution and dynamic range
- prototype with 256 tactile sensors with a spatial resolution of about 1 *mm*, addressing rows and columns as in a keyboard used 32 wires resulting in a cable diameter of less than 3 *mm*.



Optical Sensors

“frustration” of total internal reflection *UMass!*

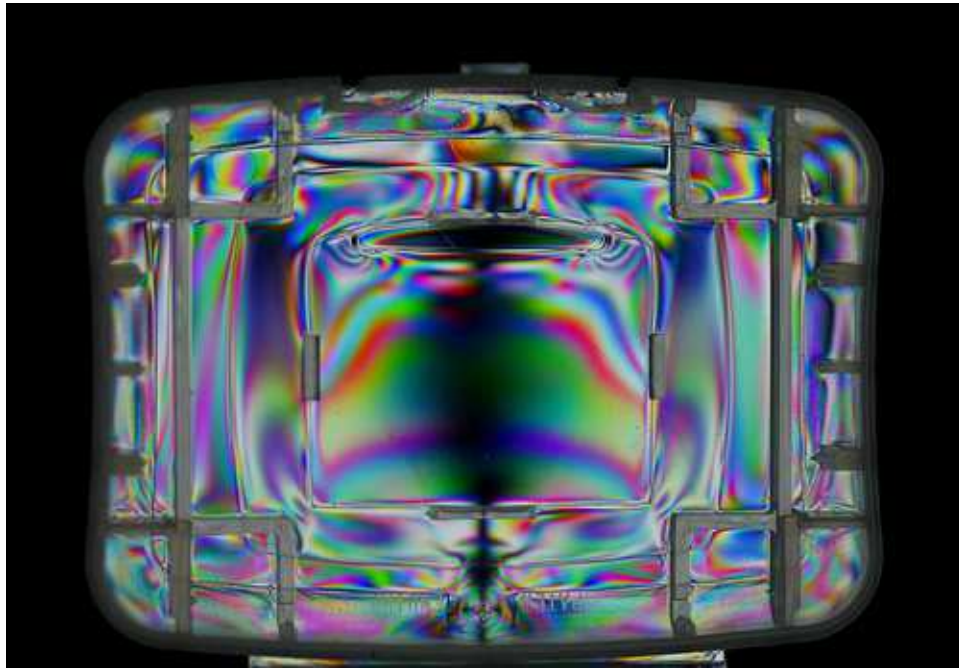


(Begej 1984)

the tactile *image* is conducted away using optical fibers and then subject to image processing



Optical Sensors - Birefringence



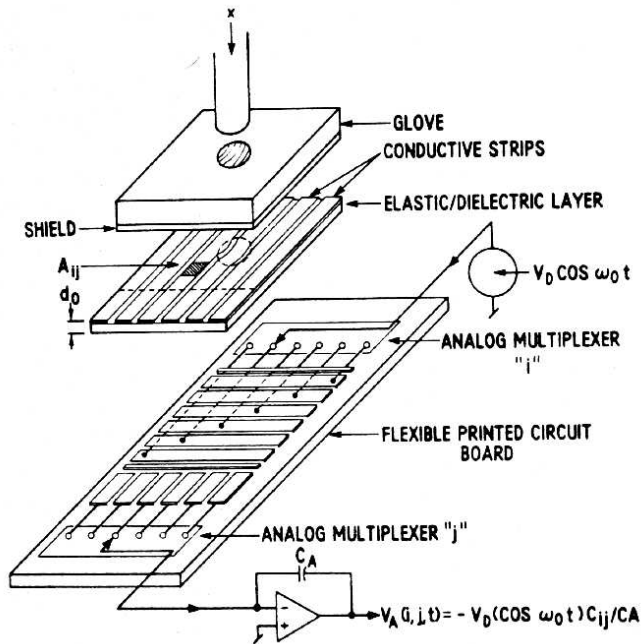
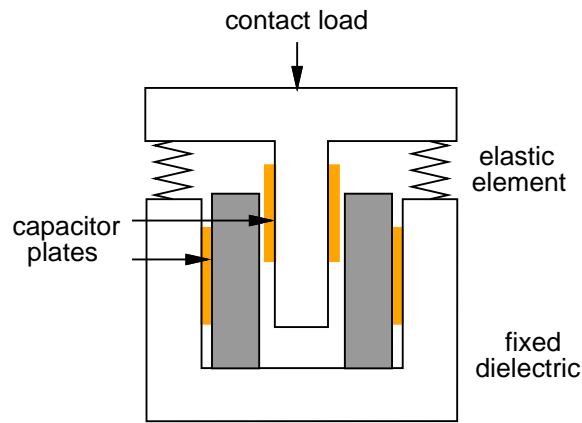
“double refraction” when light passes through anisotropic materials (calcite crystals)

isotropic solids (plastics) under mechanical stress and viewed using two crossed polarizers (transmitted light is rotated by an amount that depends on wavelength) producing chromatic spectra.



Capacitive Sensors

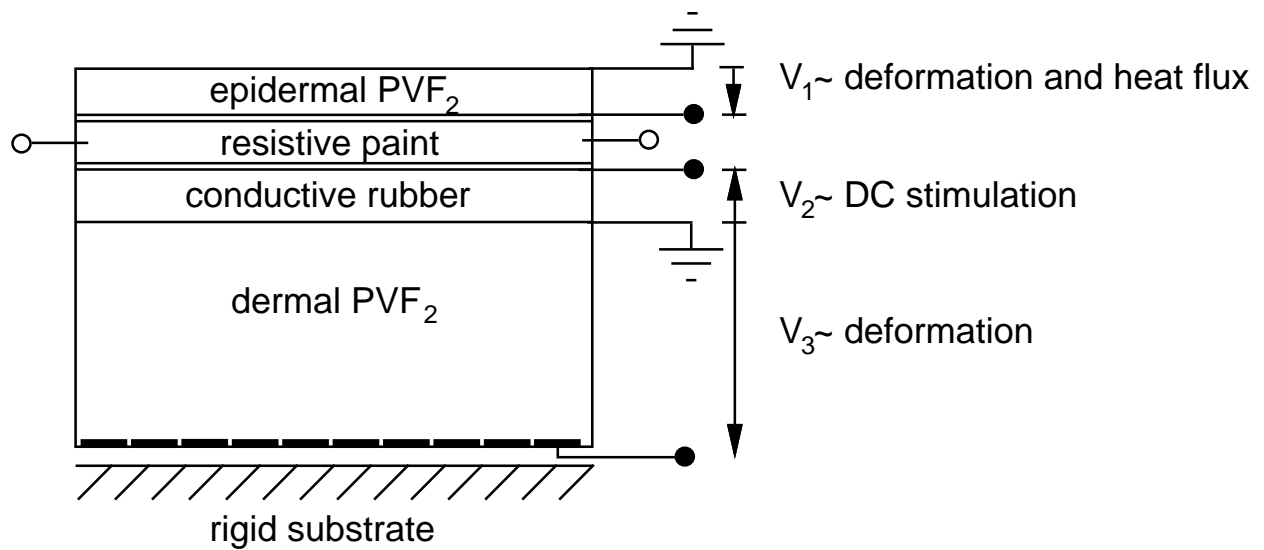
Capacitive Shutter





Piezo- and Pyroelectric Effects

PVF_2 (polyvinylidene fluoride)





Screw Nomenclature

twist: generalized velocity

$$\mathbf{v} \in \mathbb{R}^6$$

$$\mathbf{v} = \begin{bmatrix} v_x \\ v_y \\ v_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

wrench: generalized force

$$\mathbf{w} \in \mathbb{R}^6$$

$$\mathbf{w} = \begin{bmatrix} f_x \\ f_y \\ f_z \\ m_x \\ m_y \\ m_z \end{bmatrix}$$

\mathbf{v} and \mathbf{w} do not constitute linear vector spaces!

power: $\mathbf{w}^T \mathbf{v} = [f_x \ f_y \ f_z \ m_x \ m_y \ m_z] \begin{bmatrix} v_x \\ v_y \\ v_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$



Mobility and Connectivity



$\mathbf{v} \in V$: object twists consistent with contact constraints; and
 $\bar{\mathbf{v}} \in \bar{V}$: object twists that are restricted by contact constraints.

$$\text{span}\{V \cup \bar{V}\} = \mathbb{R}^6 \quad \text{and} \quad \{V \cap \bar{V}\} = \{\emptyset\}$$

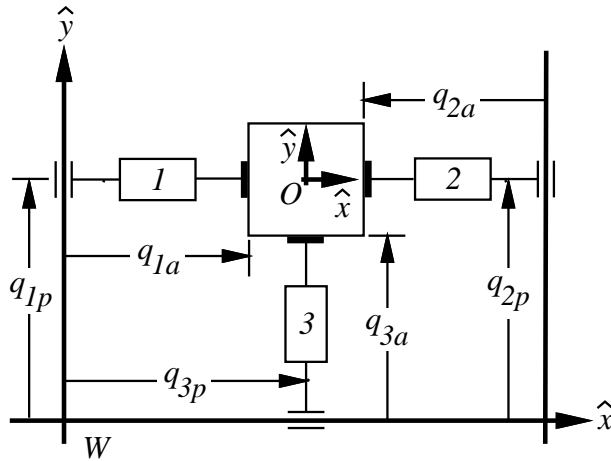
for a system of n contacts to immobilize a body:

$$\{\mathbf{v}_1 \cap \mathbf{v}_2 \cap \cdots \cap \mathbf{v}_n\} = \{\emptyset\}, \text{ and}$$

$$\text{span}\{\bar{\mathbf{v}}_1 \cup \bar{\mathbf{v}}_2 \cup \cdots \cup \bar{\mathbf{v}}_n\} = \mathbb{R}^6$$



Example: Mobility and Constraint in a Planar “Hand”



“velcro” contacts

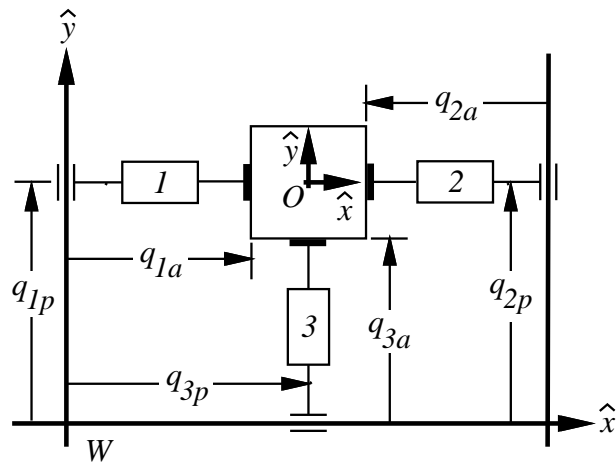
$$\begin{aligned}
 \text{finger \#1: } \begin{bmatrix} v_x \\ v_y \end{bmatrix}_O &= \begin{bmatrix} 1 \\ 0 \end{bmatrix}_O \dot{q}_{1a} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}_O \dot{q}_{1p} \\
 &= \bar{\mathbf{v}}_1 \dot{q}_{1a} + \mathbf{v}_1 \dot{q}_{1p}
 \end{aligned}$$

$$\begin{aligned}
 \text{finger \#2: } \begin{bmatrix} v_x \\ v_y \end{bmatrix}_O &= \begin{bmatrix} -1 \\ 0 \end{bmatrix}_O \dot{q}_{2a} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}_O \dot{q}_{2p} \\
 &= \bar{\mathbf{v}}_2 \dot{q}_{2a} + \mathbf{v}_2 \dot{q}_{2p}
 \end{aligned}$$

$$\begin{aligned}
 \text{finger \#3: } \begin{bmatrix} v_x \\ v_y \end{bmatrix}_O &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}_O \dot{q}_{3a} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}_O \dot{q}_{3p} \\
 &= \bar{\mathbf{v}}_3 \dot{q}_{3a} + \mathbf{v}_3 \dot{q}_{3p}.
 \end{aligned}$$



Example: Mobility and Constraint in a Planar “Hand”



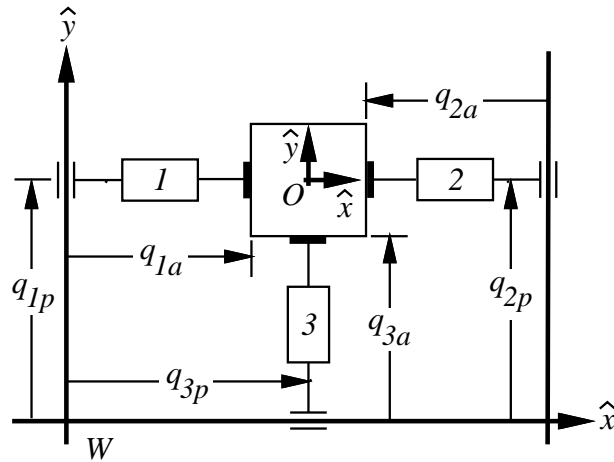
considering just fingers 1 and 2...

$$V = \bigcap_{i=1}^2 \mathbf{v}_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cap \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

⇒ fingers 1 and 2 alone do not fully immobilize the object



Example: Mobility and Constraint in a Planar “Hand”



considering fingers 1, 2, and 3, the intersection of unrestricted object velocities is empty...

$$V = \bigcap_{i=1}^3 \mathbf{v}_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cap \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cap \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \emptyset,$$

...these three (fixed) contacts fully immobilize the object,

and the union of velocity constraints derived from active degrees of freedom spans \mathbb{R}^2 :

$$\bar{V} = \bigcup_{i=1}^3 \bar{\mathbf{v}}_i = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cup \begin{bmatrix} -1 \\ 0 \end{bmatrix} \cup \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \mathbb{R}^2$$

\implies the object position fully controllable in the (x, y) plane by the planar hand.



“Form Closure” - Reuleaux (1876)

Definition (Form Closure) - a condition of complete restraint in which any object twist $\in \mathbb{R}^6$ is inconsistent with rigid body assumption for objects and fixed contacts.

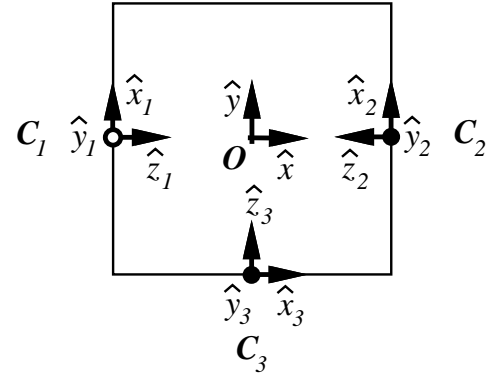
form closure can be defined solely in terms of mobility without specifying contact forces at all

- Reuleaux
 - planar bodies require at least four frictionless contacts for form closure in \mathbb{R}^3 , and
 - *exceptional* surfaces exist for which form closure is impossible given any number of frictionless point contacts.
- Somoff (1897) proved that at least 7 frictionless point contacts are necessary for form closure in \mathbb{R}^6
- Mishra, Schwartz and Sharir (1987) - established an upper bound of 6 frictionless point contacts on planar objects with piecewise smooth contours, and 12 for the spatial case (except for Reuleaux’s exceptional surfaces).



The Grasp Jacobian

contact coordinate frames
aligned with the
local surface normal



in our planar example, contact frames provide a basis for writing a linear expression mapping object twists $\mathbf{v}_O = [v_x \ v_y]^T$ into contact frame twists $\mathbf{v}_C = [v_{1x} \ v_{1z} \ v_{2x} \ v_{2z} \ v_{3x} \ v_{3z}]^T$.

$$\begin{bmatrix} v_{1x} \\ v_{1z} \\ v_{2x} \\ v_{2z} \\ v_{3x} \\ v_{3z} \end{bmatrix}_C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \end{bmatrix}_O$$

$$\mathbf{v}_C = \mathbf{G}^T \mathbf{v}_O$$

where $\mathbf{G} = [\mathbf{v}_1 \ \bar{\mathbf{v}}_1 \ \mathbf{v}_2 \ \bar{\mathbf{v}}_2 \ \mathbf{v}_3 \ \bar{\mathbf{v}}_3]$ is the **Grasp Jacobian**¹

¹or Grip Transform [?], Grip Matrix [?], or the Grasp Matrix [?].



Contact Forces

...the power transmitted to the object is equal to the power generated by the contact forces:

$$\mathbf{w}_O^T \mathbf{v}_O = \mathbf{w}_C^T \mathbf{v}_C$$

since $\mathbf{v}_C = \mathbf{G}^T \mathbf{v}_O$, we can write

$$\mathbf{w}_O^T \mathbf{v}_O = \mathbf{w}_C^T [\mathbf{G}^T \mathbf{v}_O],$$

so that,

$$\begin{aligned} \mathbf{w}_O^T &= \mathbf{w}_C^T \mathbf{G}^T, \text{ or} \\ \mathbf{w}_O &= \mathbf{G} \mathbf{w}_C \end{aligned}$$

like the manipulator Jacobian, the Grasp Jacobian captures reciprocal velocity and force mappings from contact coordinates to object coordinates.



Force Analysis in the Planar “Hand”

therefore, the Grasp Jacobian for velocity analysis also defines a transformation from contact loads $\omega_{Ci} = [f_x \ f_z]_i^T$, $i = 1, 3$ to the net wrench on the object $\omega_O = [f_x \ f_y]^T$.

$$\begin{bmatrix} f_x \\ f_y \end{bmatrix}_O = \begin{bmatrix} 0 & 1 & 0 & -1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_{1x} \\ f_{1z} \\ f_{2x} \\ f_{2z} \\ f_{3x} \\ f_{3z} \end{bmatrix}_C$$

caveats:

- velocity and force mappings using the Grasp Jacobian do not consider the (in)ability of the hand to generate velocities or forces—e.g. for the planar hand example, $f_{1x} = f_{2x} = f_{3x} = 0$
- to preserve the contact configuration, internal compressive forces are required, e.g. for the planar hand f_{1z} and f_{2z} must be strictly positive (compressive).



Contact Force Analysis

contact type	geometry	selection matrix \mathbf{H}^T $\mathbf{w}_C = \mathbf{H}^T \boldsymbol{\lambda}$	constraints
frictionless point contact		$\mathbf{w}_C = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} [\lambda_{fz}]$	$\lambda_{fz} \geq 0$
point contact with friction		$\mathbf{w}_C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_{fx} \\ \lambda_{fy} \\ \lambda_{fz} \end{bmatrix}$	$\lambda_{fz} \geq 0$ $[\lambda_{fx}^2 + \lambda_{fy}^2]^{1/2} \leq \mu \lambda_{fz}$
soft finger		$\mathbf{w}_C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda_{fx} \\ \lambda_{fy} \\ \lambda_{fz} \\ \lambda_{mz} \end{bmatrix}$	$\lambda_{fz} \geq 0$ $[\lambda_{fx}^2 + \lambda_{fy}^2]^{1/2} \leq \mu \lambda_{fz}$ $\lambda_{mz} \leq \gamma \lambda_{fz}$



Rotating Contact Wrenches

given the rotation matrix ${}^o\mathbf{R}_{C_i}$ that transforms vectors in contact frame i into object frame—the block diagonal

$$\overline{\mathbf{R}}_i = \left[\begin{array}{c|c} {}^o\mathbf{R}_{C_i} & \mathbf{0} \\ \hline \mathbf{0} & {}^o\mathbf{R}_{C_i} \end{array} \right]$$

applies this rotation to the force and moment components of the contact wrench independently.



Translating Contact Wrenches

- the force component of the wrench maps to the same forces in the object frame, and
- contact frame moments sum with the “couple” $\boldsymbol{\rho} \times \mathbf{f}_C$, where $\boldsymbol{\rho} \in \mathbb{R}^3$ is the position vector locating frame \mathbf{C} with respect to frame \mathbf{O}

$$\mathbf{P}_i = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & -\rho_z & \rho_y & 1 & 0 & 0 \\ \rho_z & 0 & -\rho_x & 0 & 1 & 0 \\ -\rho_y & \rho_x & 0 & 0 & 0 & 1 \end{array} \right]$$

the product of matrix \mathbf{P}_i with a wrench at the contact site transforms that wrench into the equivalent wrench at the object frame.



Constructing the Grasp Jacobian

$$(\mathbf{w}_O)_i = \mathbf{G}_i \mathbf{w}_{Ci} = \mathbf{G}_i \mathbf{H}_i^T \boldsymbol{\lambda}_{Ci}$$

$$(\mathbf{w}_O)_i = \mathbf{G}_i^* \boldsymbol{\lambda}_{Ci}, \text{ where, } \mathbf{G}_i^* = \mathbf{P}_i \bar{\mathbf{R}}_i \mathbf{H}_i^T.$$

For an n contact grasp configuration, the grasp Jacobian and effort is written

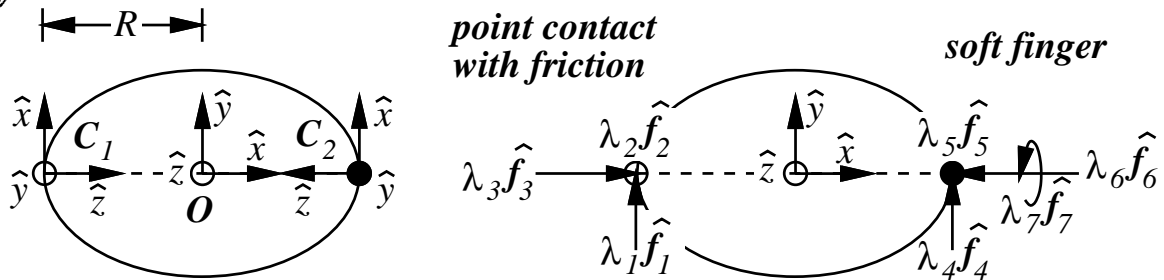
$$\mathbf{G}^* = [\mathbf{G}_1^* \cdots \mathbf{G}_n^*]$$

and,

$$\boldsymbol{\lambda} = [\boldsymbol{\lambda}_{C1}^T \cdots \boldsymbol{\lambda}_{Cn}^T]^T.$$



Solving for Grasp Forces



assume that unit contact forces, $\mathbf{f}_i \in \mathbb{R}^3$, are independent

Grasp Jacobian(by inspection) - n column vectors describing the body frame wrenches corresponding to each of the n contact forces.

$$\mathbf{w}_O = \begin{bmatrix} \mathbf{w}_1 & \mathbf{w}_2 & \mathbf{w}_3 & \mathbf{w}_4 & \mathbf{w}_5 & \mathbf{w}_6 & \mathbf{w}_7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & R & 0 & 0 & R & 0 & 0 \\ -R & 0 & 0 & R & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_7 \end{bmatrix}$$

$$\mathbf{w}_O = \mathbf{G}^* \boldsymbol{\lambda}$$



Prehensile Grasp Stability

the ability of a contact configuration to suppress random disturbance wrenches by modifying grip forces

Definition (Force Closure) - A grasp is force closure if a solution for contact frame wrenches λ exists that complies with contact type constraints such that

$$\mathbf{G}^* \lambda = \mathbf{w}_{dist} \quad \text{for arbitrary } \mathbf{w}_{dist}$$

\implies the contact configuration is capable of generating a convex envelope of grasp wrench responses (that contains the origin).
prehensile



Grasp Stability

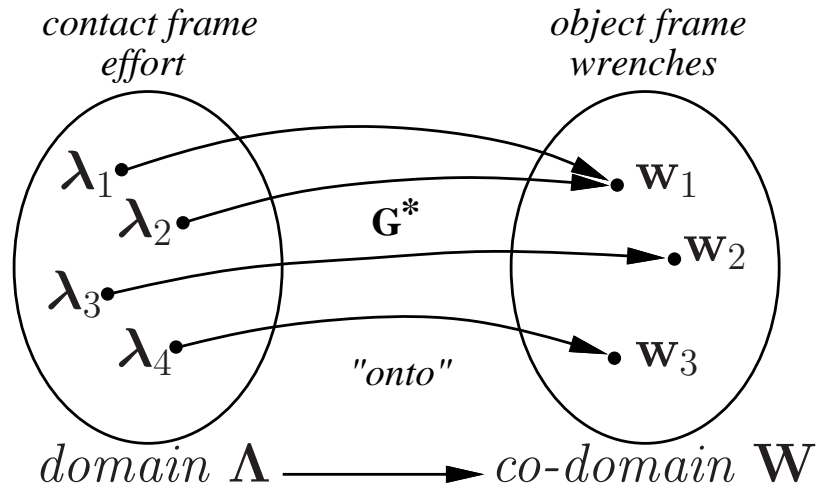
...stated in another way...

$$\mathbf{w}_O = \mathbf{G}^* \boldsymbol{\lambda}$$

a grasp is force closure (and stabilizable) if and only if \mathbf{G}^* is surjective [Murray, Li, Sastry 1994]

surjection (“onto”) - every object frame wrench \mathbf{w}_i is accessible by applying transform \mathbf{G}^* to at least one combination of contact frame effort $\boldsymbol{\lambda}$

many-to-one



$$\forall \mathbf{w}_i \in \mathbf{W} \exists \boldsymbol{\lambda} \in \Lambda \text{ such that } \mathbf{w}_i = \mathbf{G}^* \boldsymbol{\lambda}$$



Solving for Grasp Forces

$$\mathbf{w}_O = \mathbf{G}^* \boldsymbol{\lambda} = \mathbf{G}^* (\boldsymbol{\lambda}_p + \boldsymbol{\kappa}^T \boldsymbol{\lambda}_h)$$

where solutions $\boldsymbol{\lambda}$ have homogeneous and particular parts,

$$\boldsymbol{\lambda} = \boldsymbol{\lambda}_p + \boldsymbol{\kappa}^T \boldsymbol{\lambda}_h$$

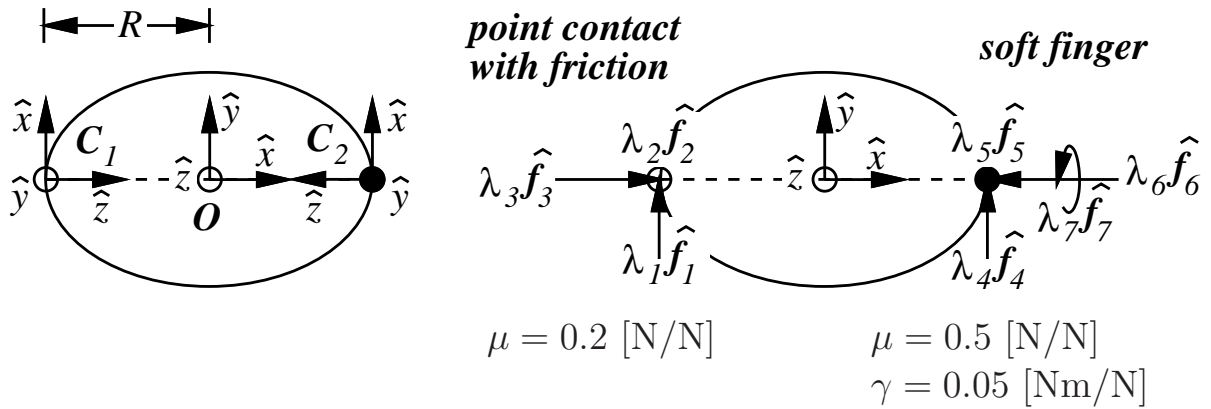
$\boldsymbol{\lambda}_h$ is the **homogeneous part** of the solution and describes combinations of contact forces that impart zero net force to the object.

$$\mathbf{G}^* \boldsymbol{\lambda}_h = \mathbf{0}$$

- \mathbf{G}^* must be full rank to achieve arbitrary reference wrenches
- $\boldsymbol{\lambda}$ must satisfy inequality constraints for unisense normal forces and contact friction.



Solving for Grasp Forces



suppose: grasp forces must support an object load of $-1.0\hat{y}$ [N]

$$M_x = 0 \Rightarrow \lambda_7 = 0$$

$$F_y = 1 \Rightarrow \lambda_1 + \lambda_4 = 1$$

$$F_x = 0 \Rightarrow \lambda_3 - \lambda_6 = 0$$

$$M_z = 0 \Rightarrow -\lambda_1 + \lambda_4 = 0$$

$$\Rightarrow \lambda_3 = \lambda_6$$

$$\Rightarrow \lambda_1 = \lambda_4 = 0.5$$

frictional constraints

$$F_z = 0 \Rightarrow \lambda_2 - \lambda_5 = 0$$

$$\lambda_1 \leq \mu\lambda_3$$

$$M_y = 0 \Rightarrow \lambda_2 + \lambda_5 = 0$$

$$0.5 \leq (0.2)\lambda_3$$

$$\Rightarrow \lambda_2 = \lambda_5 = 0$$

$$\lambda_3 \geq 2.5$$

$$\lambda = \lambda_p + \kappa^T \lambda_h = [0.5 \ 0 \ 0 \ 0.5 \ 0 \ 0 \ 0]^T_p + \kappa [0 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0]^T_h$$

and, $\kappa \geq 2.5$ satisfies frictional constraints

automated techniques based on mathematical programming are used to solve these systems subject to contact type constraints



Force Closure - revisited

1. The grasped object is in quasistatic equilibrium, there are no net forces or moments,
2. all forces are applied within the cone of friction so that there is no slippage, and,
3. an externally applied force can be resisted by finger forces with a finite and controllable deflection.



Grasp Planning

1. Salisbury (1982) - analytical framework for evaluating grasp stability in which the stiffness matrix that characterizes a grasp must be positive definite.
2. Cutkosky (1985) - grasp stability depends on force distributions and local curvature
3. Montana (1988) - contact grasp stability evaluates the ability of a perturbed grasp geometry to return to an equilibrium configuration—kinematic description of rolling contacts
4. Nguyen (1989) - all force closure grasps are *stabilizable* by actively modulating contact forces.
5. Hemami (ca. 1989) - treated dynamic stability using the methods of Lyapunov.
6. Ferrari (1992) - grasp metrics for judging the quality of a grasp for planning methods.
7. Bicchi (1995) - tested force closure and a quality metric for a grasp given friction, contact forces, and constraints on applied forces.
8. Coelho (1994-01), Platt (2002-06) - haptic feedback for grasp control.
9. Ng (2006) - associating visual features with grasp postures.